18.02 ρ Problem Set 9

(Due Tuesday, December 6, 11:59:59 PM)

Part I (54 points)

HAND IN ONLY THE UNDERLINED PROBLEMS

(The others are *some* suggested choices for more practice.) EP = Edwards and Penny; SN = Supplementary Notes (most have solutions)

Divergence Theorem

Reading: EP §15.6 SN §V10 Exercises: EP §15.6 4, 5, $\underline{7}$, $\underline{9}$, 10, 12, $\underline{16}$, $\underline{17}$, $\underline{21}$, $\underline{23}$, $\underline{27}$ SN §6C $\underline{4}$, 5, $\underline{7}$, 11

Part II (13 points)

Directions: Try each problem alone for 20 minutes. If you collaborate later, you must write up solutions independently.

Problem 1 (20) Consider a torus of inner radius 3, outer radius 5 whose plane of symmetry is the *xy*-plane and whose center lies at the origin. Let S_1 be the part this torus above the *xy*-plane and let S_2 be the portion of the *xy*-plane lying inside this torus. Let $\mathbf{F}(x, y, z) = (x + \sin(yz), x^2(2yz - 1) + z^2, x^2 + y^2 + z^2 - z)$. Compute $\int_{S_1} \mathbf{F} \cdot \mathbf{n} dS$ by computing $\int_{S_2} \mathbf{F} \cdot \mathbf{n} dS$ and then using the divergence theorem. [Hint: what type of symmetry does the torus have?]