Practice Answers for Final
18.02 ESG Spring 2005

1. This is a cone, vertex at the origin, axis (1,1,1) and vertex half-angle $\cos^{-1}(\frac{\sqrt{3}}{3})$.

2. The curve is given by $(25t^2 + 4t - 3t^2 \sin t - 6\cos t - 2, t^3 + 2\cos t + 2t\sin t + 4\sin t - 27, 2t^2 + 24t + \sin t)$.

3. The parametric equations are $(\frac{13}{45} + \frac{1}{7}t, \frac{52}{45} + \frac{67}{7}t, \frac{209}{45} + \frac{23}{7}t)$. You can find the symmetric equations from this.

4. CHALLENGE PROBLEM (I think this problem is cool. By no means are you expected to know how to do this.) Since you don't need to know how to do this I'm not going to give you the answer. I'll tell you later.

5. 3

6. $(\pm 2, 0, 4)$ and $(\pm 1, -1, \frac{5}{2})$.

7. The derivative is
$$A = \begin{bmatrix} 4 & 1 \\ 1 & -12 \\ 14 & 13 \end{bmatrix}$$
This is the matrix such that $f(x,y) \cong A \cdot \begin{pmatrix} x-1 \\ y-2 \end{pmatrix} + f(1,2)$.

8. (0,0) is a saddle, $(3 \cdot (405)^{\frac{2}{7}}, -(405)^{\frac{1}{7}})$ is a minimum.

9. The gradient is the matrix derivative of the map and thus f can be approximated locally as just the dot product with the gradient. Thus the level surface, locally, is the set of vectors perpindicular to the gradient.

10. The line in the xy-plane with equation $48(x-2) - 24(y-4) = 16$.

11. $\frac{\sqrt{6}}{3}$.

12. $(0, 0, \frac{16}{3})$.

13.

$$\int_{-2}^{2}\int_{-\sqrt{1-\frac{x^2}{4}}}^{\sqrt{1-\frac{x^2}{4}}}\int_{-\sqrt{4-x^2-4z^2}}^{\sqrt{4-x^2-4z^2}} dy\, dz\, dx - 2\int_{-\frac{3}{4}\sqrt{6}}^{\frac{3}{4}\sqrt{6}}\int_{-\sqrt{\frac{108-32x^2}{135}}}^{\sqrt{\frac{108-32x^2}{135}}}\int_{\sqrt{1-\frac{z^2}{4}-\frac{x^2}{9}}}^{\sqrt{4-x^2-4z^2}} dy\, dz\, dx$$

14. A is invertible.

15. $40\sqrt{2}$

16. $\frac{15}{2}$.