Research Statement

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I study **computational number theory**. Specifically, I focus on **building mathematical databases** and **p-adic computation**. I see computational efforts as being complementary to the more traditional research output consisting of papers in journals. Just as a theorem can serve as a key ingredient in another mathematician's work, software and online databases enable exploration of conjectures and familiarization with new mathematics as well as computational contributions to the proofs of theoretical results. I prioritize making my research broadly accessible to the mathematical community by contributing to open source projects, posting papers and code publicly, and organizing workshops to support other mathematicians' use of computational tools.

I have divided this statement into two parts, with the first focusing on databases and the second on p-adic computation. In the database section I summarize ongoing work within the L-functions and Modular Forms Database (LMFDB) on finite groups, modular forms, modular curves and abelian varieties over finite fields. The second section is more focused topically but more diverse in terms of investigative methods. I describe my implementation of p-adic arithmetic in Sage [Wil05] together with a comparison of tracking computational precision in the real and p-adic settings. Afterward, I give more details on computational challenges arising when working with p-adic extension fields, including a discussion of Galois groups and ramification. I then explain an ongoing application of p-adic computation to arithmetic geometry, with a new algorithm for computing L-functions of hypergeometric motives over $\mathbb{Q}$. Finally, I close with a speculative long term project: a p-adic version of the Atlas of Lie Groups and Representations.

1 Databases

The LMFDB [LMF23a] is a website that hosts dozens of collections of mathematical objects, from the eponymous L-functions and classical modular forms to elliptic curves, number fields, finite groups and Artin representations. Mathematically, it is motivated by the Langlands program, but many of the objects arise much more broadly.

These databases make a new kind of experimental mathematics possible, with a spectacular example occurring last year. While studying elliptic curves ordered by conductor, a group of mathematicians including an undergraduate student [HLOP22] observed that the average point counts modulo primes displayed an unexpected oscillation. This "murmurations" phenomenon was only apparent when averaging over tens of thousands of curves, and has led to further theoretical exploration including a workshop at ICERM in July 2023, a proof of an analogue in the modular form setting, and investigations in non-arithmetic settings like Maass forms. As the use of data more broadly in society continues to grow, I believe that mathematical databases have a key role to play.

I have been involved for over a decade, contributing both to improving the underlying infrastructure [CR21a] across the whole site and to creating and expanding the component databases [DKRV21, BBB$^+$21, CJP$^+$23]. The LMFDB has developed into an important resource for number theorists, with over 700 academic citations on Google Scholar and over 5000 monthly visits. I encourage you to take a break from this proposal and explore the website.

Work on building databases forms a core part of my research output. Each section of the LMFDB provides a search interface, allowing researchers to find examples and counterexamples as they study these objects. There are also easy ways to browse interesting examples, as well as extensive exposition in expandable "knowls," allowing novices to learn about the underlying mathematics without cluttering the interface for more advanced users. In addition to providing examples, we aim to give input ranges for which we can guarantee that there are no missing objects. Such a guarantee can be crucial when applying the database in a proof; for example, one ingredient in the proof of Fermat's Last Theorem is that there are no modular forms of weight 2 and level 2.

1.1 Finite groups

The database of finite groups in the LMFDB [CJP+23] was unveiled in July 2023, though more work is ongoing. It provides a searchable database of over 500,000 groups [LMF23c] that are small in some way: either with small cardinality (from the SmallGroup database in GAP and Magma), with a small permutation representation (abstract isomorphism classes of groups from the transitive group database), arising as a matrix group in small dimension over a ring such as $\mathbb{Z}$, $\mathbb{F}_q$ or $\mathbb{Z}/N\mathbb{Z}$, or with a short composition series (simple, perfect and almost simple groups). For each group, the subgroup lattice, character table, and other properties are stored, to the extent that these computations are feasible. One of the main improvements that it offers over previous group databases is that it includes subgroup and quotient relationships rather than just information on how to construct each group.

I have been highly involved in this effort over the last three years, working on writing Magma code and structuring the computations to gather group theoretic invariants as completely as possible. I am excited about this database because it offers a qualitative difference with the rest of the LMFDB in terms of its connections with areas of mathematics outside number theory. Because finite groups arise so ubiquitously, there is room for cooperation with researchers studying graph theory, error correcting codes, quantum computation and even crystallography and molecular orbitals in chemistry.

Because finite groups occur much earlier in the undergraduate curriculum than the other parts of the LMFDB, this work opens up a lot of opportunities for research projects with students. I supervised two such projects in the summer of 2022, one on studying finite subgroups of $\mathrm{PGL}_n(\mathbb{C})$ for small n and the other on finding the smallest degree of a permutation representation of an abstract group. Some other possible projects for undergraduates include:

1. Add Brauer character tables for groups in the database, complementing the existing ordinary character tables;
2. Building on the databases of Galois groups and number fields, create a database of nontransitive permutation groups and a database of étale algebras;
3. Devise algorithms to improve the layout of subgroup diagrams and implement features to better explore these diagrams, which can get unwieldy for larger groups;
4. Add more families of groups (like generalized dihedral groups) and build connections to existing databases of graphs;
5. Add labels for crystallographic groups and build connections to symmetry groups in inorganic chemistry.

1.2 Modular forms

Modular forms have played a central role in number theory over the last several decades, from the proof of Fermat's last theorem and subsequent applications to Diophantine equations to connections with quadratic forms and automorphic representations. For a positive integer k and finite index subgroup $\Gamma \subseteq \mathrm{SL}_2(\mathbb{Z})$, the space of modular forms of weight k and level Γ is finite dimensional. Each and has a basis whose Fourier coefficients provide a connection to arithmetic geometry and L-functions. In 2019, I helped build the most extensive existing database of modular forms [BBB+21]. It currently contains 281,965 newforms [LMF23e], corresponding to 14,417,694 modular forms over $\mathbb{C}$. In addition to covering a broader range of weights and levels than previous databases, it made great strides in enumerating weight 1 modular forms, where modular symbol algorithms are not applicable. As part of our effort to ensure the reliability of the data, I designed and implemented a test suite for the database that ran extensive internal and external consistency checks.

1.3 Modular curves

The modular forms database played a key role in the creation of a new database of modular curves [LMF23d]. I helped organize two workshops at MIT in 2022 to kick off the creation of this database, and then worked to put together contributions from the 46 participants in those workshops and incorporate it into the LMFDB.

An elliptic curve E can be specified by an equation of the form $y^2 = x^3 + ax + b$; they are special because the set of points (x, y) on an elliptic curve forms a group. The set $E(\mathbb{Q})$ of rational points on E is a finitely generated abelian group, and a theorem of Mazur identifies the possible orders of elements.

Modular curves parameterize elliptic curves together with some extra structure. For example, given a positive integer N and any field K of characteristic not dividing N, K-points on the classical modular curve $Y_1(N)$ correspond to an elliptic curve E together with a point $P \in E(K)$ of order N. Mazur's theorem can be translated as saying that $Y_1(N)$ has a rational point if and only if $1 \leq N \leq 10$ or $N = 12$.

Beyond $Y_1(N)$, there are many modular curves, each parameterizing elliptic curves whose associated Galois representation has a certain image. Making this precise requires a short technical interlude. To any elliptic curve E over $\mathbb{Q}$, the group of N-torsion points $E[N]$ will be isomorphic to $(\mathbb{Z}/N\mathbb{Z})^2$. These points are usually not rational, but rather lie in $\overline{\mathbb{Q}}$, so the absolute Galois group $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ acts on $E[N]$, and taking a limit over N we get a representation $\rho_E : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GL}_2(\hat{\mathbb{Z}})$. Serre's open image theorem [Ser72] implies that for most elliptic curves (any curve with endomorphism ring $\mathbb{Z}$), the image of ρ_E has finite index. Given any such subgroup $H \subseteq \mathrm{GL}_2(\hat{\mathbb{Z}})$, we may define a modular curve X_H whose points parameterize elliptic curves E so that $\rho_E \subseteq H$ up to conjugacy. Each such X_H has three basic invariants:

1. the level N, so that H is equal to the full preimage of its reduction modulo N,
2. the index i of H inside $\mathrm{GL}_2(\hat{\mathbb{Z}})$,
3. the genus g of X_H as a curve.

The modular curve database in the LMFDB contains all curves with level up to 70, models (explicit equations) for many curves of genus up to 24, and rational points coming from the LMFDB's databases of elliptic curves (the Galois images were computed using an algorithm of Zywina [Zyw22] as part of this project).

Work on this database is still ongoing. We aim to include modular curves of higher level, to search for points using the explicit models (hopefully finding new examples of elliptic curves with interesting Galois representations), to add quotient curves under Atkin-Lehner operators, and to add automorphism groups and exceptional isomorphisms. In addition, we hope to connect this database to the extensive literature on modular curves so that it can serve as a dynamic and growing resource for the community.

1.4 Abelian varieties over finite fields

While many interesting questions about elliptic curves remain open, a lot of focus in computational number theory has shifted in recent years to higher genus curves (elliptic curves are genus 1 curves equipped with a base point). One of the central objects of study for a curve of genus g is its Jacobian, defined in terms of formal sums of points on the curve. Jacobians are examples of abelian varieties, higher dimensional analogues of elliptic curves that are simultaneously projective varieties and abelian groups. For $g > 2$, not all abelian varieties are Jacobians; the characterization of Jacobians among all abelian varieties is known as the Schottky problem.

Abelian varieties were originally studied over the complex numbers, but arithmetic geometers prefer to work over number fields and finite fields. Over a finite field $\mathbb{F}_q$, the Honda-Tate theorem [WM71] provides a powerful tool for studying abelian varieties. It classifies them up to isogeny, which is an equivalence relation determined by the presence of a homomorphism with finite kernel, and gives a bijection[1] between

[1] the image needs to be adjusted slightly by requiring that certain irreducible factors occur with multiplicity

isogeny classes over $\mathbb{F}_q$ of dimension g and integer polynomials of degree $2g$ whose roots in $\mathbb{C}$ all have absolute value $\sqrt{q}$. Using this correspondence, Taylor Dupuy, Kiran Kedlaya, Christelle Vincent and I built a database of abelian varieties [DKRV21], including quantities like point counts, endomorphism algebras, twists, primitivity, and angle ranks that are isogeny invariant and can be determined from the corresponding polynomial.

The current database [LMF23b] contains almost 3 million isogeny classes, of dimension up to 6. I have been working in two directions to enhance it. First, Stefano Marseglia and Mckenzie West have joined us and we are making progress on dividing isogeny classes up into isomorphism classes. This process proceeds in two steps, working first with unpolarized abelian varieties and then computing polarizations (a polarization of A is an isomorphism from A to its dual abelian variety). Given an isogeny class corresponding to a polynomial $f(x)$, the polynomial defines an order[2] and, under certain constraints on f, unpolarized isomorphism classes correspond to ideal classes in this order. Polarizations and explicit isogenies can be computed in terms of the same data. We have completed a draft of the resulting dataset, and aim to include it in the LMFDB in the next several months.

The second direction relates to the Schottky problem. When $g > 2$, there is no known method for determining whether a single isogeny class (specified by a polynomial) contains a Jacobian. Instead, various authors have enumerated all curves of genus g over $\mathbb{F}_q$ for g up to 5 and small values of q. I am working with Kiran Kedlaya on using descriptions of genus 6 and 7 curves [Muk95] to carry out a similar project in higher genus. Longer term, enough of this form may make it possible to use machine learning algorithms to search for a pattern in which polynomials correspond to isogeny classes containing a Jacobian.

2 p-adic computation

p-adic numbers have played a central role in many advances in modern number theory, from Deligne's proof of the Weil conjectures to the deformation theory underlying Wiles' proof of Fermat's Last Theorem to Scholze's perfectoid spaces. They can be defined either analytically, via the completion process that constructs the real numbers from the rationals, or algebraically, as the inverse limit of finite rings $\mathbb{Z}/p^n\mathbb{Z}$ as n increases. In either perspective, they provide a link between divisibility and modular arithmetic on the one hand and topology and analysis on the other.

My interest in p-adic computation began as an undergraduate, with a project that used p-adic cohomology to count points on surfaces over finite fields [AKR09] and a project that studied the Coleman-Mazur eigencurve [CM98] in the 3-adic setting [Roe14]. My contributions developed in a more theoretical direction with my thesis on the local Langlands correspondence for tame unitary groups [Roe11], two papers with Clifton Cunningham on a function-sheaf dictionary for characters of group schemes [CR16, CR21b], and a paper with Moshe Adrian on rectifiers in the local Langlands correspondence [AR16]. These projects have informed my understanding of how p-adics can be applied to computational problems and provided motivation for studying p-adics in representation theory, which I will discuss further in Section 2.4.

From a computational point of view, there are two main challenges in implementing p-adic arithmetic: tracking precision through a computation, and handling algebraic extensions, which are far more complicated than for real numbers.

2.1 Tracking precision

Both real and p-adic arithmetic can only be implemented to finite precision on a computer, but error tracking is easier in the p-adic context because p-adic fields are non-Archimedean. For example, arbitrarily many inexact values can be added and the result will have the same precision as the least precise input. Yet once addition and multiplication are mixed, precision loss occurs both in theory and practice. Partly as a consequence of these non-Archimedean advantages, far less work had been done in the p-adic setting than over the real numbers, where numerical methods are commonly studied. In a sequence of papers with

[2] a subring of full rank inside the number field defined by the same polynomial

Xavier Caruso and Tristan Vaccon [CVR14, CVR15, CVR16, CVR17, CVR18], we explored a new method for propagating precision bounds through a computation, based on the following foundational lemma.

Let V and W be vector spaces over $\mathbb{Q}_p$ of dimension m and n. The p-adic ball $B_V(r)$ of radius $r = p^{-a}$ around 0 in V is a lattice, $p^a\mathbb{Z}_p^m$. More general ellipsoids can be modeled as arbitrary lattices $H \subset V$, and an imprecise element of V can be modeled as a coset $v + H$.

Lemma 1 ([CVR14, Lem. 3.4])**.** *Suppose that $f : V \to W$ is differentiable at $v \in V$ and that the differential $f'(v)$ is surjective. Then, for all $\rho \in (0, 1]$, there exists $\delta \in \mathbb{R}_{>0}$ such that, for all $r \in (0, \delta)$ and all lattices H with $B_V(\rho r) \subset H \subset B_V(r)$ one has*

$$f(v + H) = f(v) + f'(v)(H).$$

The unique feature of this lemma is the *equality* of $f(v + H)$ and $f(v) + f'(v)(H)$, which implies that tracking precision through lattices and differentials is *optimal*, since the image of $v + H$ is also given by a lattice. Accompanying the final paper in this series, we implemented lattice precision within Sage, making it broadly and easily usable by other mathematicians.

The main downside to the method is that the complexity grows dramatically with the dimension, when compared to simpler methods. There is a lot of room for student projects that use the lattice approach to model precision loss theoretically for specific problems, while designing numerically stable algorithms for computing approximations without precision tracking.

2.2 Extensions and Galois groups

Unlike $\mathbb{R}$, which has a unique nontrivial algebraic extension (the complex numbers), any p-adic field $\mathbb{Q}_p$ has infinitely many extensions (see [Ser79] for example). To any such extension $K/\mathbb{Q}_p$ we may associate its Galois group $G = \mathrm{Gal}(K/\mathbb{Q}_p)$, a permutation group that controls how K is situated among other extensions $L/\mathbb{Q}_p$. For example, for each odd prime p there is a unique quartic extension with Galois group C_2^2, and it contains all the quadratic extensions of $\mathbb{Q}_p$. Moreover, there is a natural filtration on G (the ramification filtration), and corresponding sequences of subfields of K and of the normal closure of K. There are several computational problems naturally arising in this context:

1. Given K, compute G and the ramification filtrations on G and K,
2. Given G, find all K with $\mathrm{Gal}(K/\mathbb{Q}_p) \cong G$ (there are finitely many),
3. Build a database of extensions $K/\mathbb{Q}_p$, giving a canonical defining polynomial for each K.

The problem of computing Galois groups has been well studied for number fields, and an efficient algorithm in Magma [BCP97] actually uses unramified extensions of $\mathbb{Q}_p$ internally. Ironically, the case of $K/\mathbb{Q}_p$ is more difficult because there is no flexibility in the choice of p. For some ramified extensions, any splitting field has extremely large degree, making Stauduhar's method prohibitively expensive. There have been advances due to Doris [Dor20], and Milstead [Mil17] (both PhD students) but many degree 32 extensions of $\mathbb{Q}_2$, for example, remain out of reach. Current approaches to computing ramification filtrations, either in the number field of local field setting, require computing in a p-adic splitting field and are thus also restricted to quite low degree.

For the second question, I gave an algorithm [Roe19] for counting the number of K with a given Galois group, as long as $p \neq 2$. There are some natural necessary conditions on G for such a K to exist, but no known, easy-to-state sufficient conditions. I am currently working with Jordi Guardia, John Jones, Kevin Keating, Sebastian Pauli, and David Roberts on a collaboration (through the SQuaRE program at AIM) where we hope to build on work of Monge [Mon14] to understand how Galois groups vary in families of Eisenstein polynomials, as well as improve the Jones-Roberts database [JR06] within the LMFDB. We are making progress on choosing a canonical polynomial defining each K; the analogous function for number fields, `polredabs` in Pari [BBB+85], has proven very useful since it enables looking up an unknown number field from a list of existing fields.

As with finite groups, this material is relatively accessible for undergraduate and graduate students. Some possible student projects include:

1. Continuing work on algorithms for computing Galois groups of p-adic fields;
2. Find a description of the absolute Galois group of $\mathbb{Q}_2$ as a profinite group (this has been done for $p > 2$ as well as for many extensions of $\mathbb{Q}_2$, but not for $\mathbb{Q}_2$ itself).

2.3 Hypergeometric L-functions

Hypergeometric motives are a class of motives that provide an avenue to constructing a wide variety of L-functions that are not easily accessible via direct computation from an explicit algebraic variety. They are defined in terms of very simple data: a rational function $f(T)/g(T)$ so that f and g have equal degree and can be expressed as products of $T^m - 1$ for varying m, together with a specialization parameter $t \in \mathbb{Q}$. A full definition is given in the survey [RR22], and the Dirichlet coefficient a_p of the associated L-function can be expressed [Wat15] as an explicit sum with p summands. Computing the L-function to precision N using this formula directly thus requires $O(N^2)$ operations. Together with Edgar Costa and Kiran Kedlaya [CKR20, CKR23], we have designed an implemented an algorithm that uses methods originally due to Costa, Gerbitz and Harvey [CGH14] to compute the L-function in $\widetilde{O}(N)$ operations.

The performance impact is dramatic in practice, opening up the possibility of computing many hypergeometric L-functions. Such a source of L-functions has two immediate applications: examples are important in pinning down Euler factors at wild primes in order to get a full conjecture on what the conductor of a hypergeometric L-function should be, and a large body of high degree L-functions will be valuable in determining how widely the new murmurations phenomenon [HLOP22] applies.

2.4 A p-adic Lie Atlas

The Atlas of Lie Groups and Representations [Atl23] created software for computing unitary representations of real and complex reductive groups. Their computation of Kazhdan-Lusztig-Vogan polynomials for E_8 drew a lot of attention, with articles in the New York Times and BBC. Carrying out a similar project for p-adic groups is a natural next step, but there are many obstacles.

The first step is a computational understanding of algebraic tori, how they fit inside p-adic reductive groups, and software to compute their Weyl groups and Moy-Prasad filtrations. A classification of p-adic reductive groups is described by Tits [Tit79], though implementing this computationally and designing an interface remains a challenge. There are several approaches to studying algebraic tori, but perhaps the most powerful is via the equivalence of categories between tori of dimension d over a field k and representations of $\mathrm{Gal}(\overline{k}/k)$ on rank-d $\mathbb{Z}$-modules. This leads to the following approach for enumerating algebraic tori of dimension d over $\mathbb{Q}_p$:

1. List the finite subgroups of $\mathrm{GL}_d(\mathbb{Z})$ up to conjugacy,
2. For each subgroup G, find all extensions $K/\mathbb{Q}_p$ with $\mathrm{Gal}(K/\mathbb{Q}_p)$ isomorphic to G,
3. For each G and each possible K, list the isomorphisms $G \cong \mathrm{Gal}(K/\mathbb{Q}_p)$ up to conjugacy within $\mathrm{GL}_d(\mathbb{Z})$.

Note that for a given d there are only finitely many p for which there exist tori over $\mathbb{Q}_p$ that are not tamely ramified, and for a given d and p there are only finitely many tori of dimension d over $\mathbb{Q}_p$ up to isomorphism. My work on databases of finite groups and p-adic fields has been motivated partly by this application.

Of course, even after building up the algebraic group infrastructure, there are many problems in representation theory ahead. I intend to gather other researchers interested in this pursuing such a project in order to bring to p-adic groups the same kind of tools that the Atlas project brought to real groups. Once such an effort gets off the ground, I anticipate it being a source of projects accessible to graduate students as well. For example, Kaletha's description of supercuspidal L-packets [Kal19] is explicit, but software or a database making it more accessible would be very helpful.

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