

SEMINAR IN NUMBER THEORY: THE TOPIC PROPOSAL
MIT 18.784, SPRING 2026

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1. ASSIGNMENT

To culminate the undergraduate seminar on arithmetic dynamics, you will write a 8-to-12 page term paper in the style of a research or expository mathematical journal article. The goal of the term paper assignment is for you to learn and synthesize a topic of your choosing and present it in a way that is accessible to your classmates. The writing process will be broken down into stages to help you fine-tune every aspect of the paper.

First, you will write a topic proposal. Choose a focus for your term paper that interests you and prepare a shortlist of main academic references (e.g. journal articles, arXiv preprints, books, theses). Plan how you will use each reference for your paper and what additional perspective you will provide yourself.

Write your topic proposal in \LaTeX using the *amsart* class (a template is available on the course website). It should be between 1 full page and 3 full pages in length, and contain the following:

- the proposed topic in arithmetic dynamics;
- a list of *at least two* main academic references;
- a description of the information that you plan to use from each reference;
- a description of the perspective that you plan to provide yourself.

Upload the PDF of your topic proposal to your course Google Drive folder (or email it to me) by the due date listed on the course website. The following sections explain how to pick a good topic. At the end is a list of possible ideas for topics.

1.1. Reminder on AI usage. The usage of artificial intelligence (AI) tools are not permitted for this assignment, EXCEPT 1) to generate ideas during the topic proposal phase and 2) to find \LaTeX commands. In particular, no part of your topic proposal (including references) should contain any directly AI-generated writing or paraphrased AI-generated writing.

Please refer to the course syllabus for the course policy and potential consequences regarding plagiarism and AI usage.

2. FOCUS AND BREADTH

When choosing your topic should be, the following points on breadth and scope can be helpful.

- **Choose a topic that aligns with your interests.** A topic that you are passionate about will make your writing more engaging.
- **Choose a topic that aligns with your knowledge.** As you may have experience already when preparing your talks this semester, some of your sources will assume background with which you or your audience may be

unfamiliar. You can dedicate parts of your paper to reviewing this background (in fact, you probably should), but don't underestimate the task of understanding your sources *and* distilling the key ideas down into an accessible short paper.

- **A broader topic generally requires a longer paper to cover it adequately.** Since your paper is limited to roughly 10 pages, a narrower, more focused topic will allow for deeper exploration. Also consider the space you will need to dedicate to introducing background material.
- **Decide on the *purpose* of your paper, as this affects its scope.** Is it a comprehensive overview of a specific conjecture or theorem? Is it a detailed investigation on how a particular technique (e.g. from topology or analytic number theory) can be applied to arithmetic dynamics?
- **Consider the perspective that you will provide.** Your paper should have value for your readers beyond what's in the original sources. You contributing something to the greater body of mathematical literature!

Example 1 (An interesting theorem). You might choose to focus on a single theorem or two closely related theorems that you want to understand. You could gain an understanding of the theorem by reading two or more sources that present different proofs of the theorem, examples, and/or applications. Choose the proofs and examples that you think are the clearest and most helpful. Then in your own words, explain those proofs, examples, and applications to your audience (your peers) so that they can understand the theorem too, taking care to “fill in the gaps” for your audience (your peers) on the background that the original sources might assume from their intended audience (researchers). Even though you use your own words, be sure to cite the source of each proof, example, and application you present.

Warning 1 (Summarizing a reference). You might choose to focus on a single theorem, such as the classification of rational periodic points of quadratic polynomials of period 3. You may be tempted to read about the theorem from the reference [HM15] and then simply summarize its presentation in your paper. This is a poor choice because if you simply summarize, then you have not added your own perspective; there is no reason for readers to read your paper instead of reading the original source [HM15]. Furthermore, if your presentation is close enough to the original, then you may be committing plagiarism (even if you are paraphrasing in your own words).

Warning 2 (Overly broad focus). You might choose to write about a general object or concept, such as “transcendental numbers in Diophantine approximation”. You may be tempted to write a paper that presents everything you know about how transcendental numbers can be approximated, how that may differ from the approximation of algebraic numbers, and how Diophantine approximation results can prove transcendence. It's certainly possible to write about several results and applications in a cohesive way, but if there isn't a pervasive central theme then your paper may benefit from a more narrow focus. If you include several different theorems and applications that are largely *unrelated to each other*, then it can be very difficult to incorporate a logical structure and you may lose your readers in the disconnected areas.

3. EXAMPLES OF SIMILAR PAPERS

Consider reading a few examples of similar articles before finalizing your choice. [MathSciNet](#) is a great resource for finding published papers on specific topics and contains short reviews of most published works in mathematics. If you need access to MathSciNet off-campus, find it through [MIT Libraries](#).

The following journals specialize in publishing well-written articles that are typically 5 to 20 pages long.

- *The American Mathematical Monthly*
- *The Mathematical Intelligencer*
- *Mathematics Magazine*

Here are a few examples related to Diophantine approximation: [[Bur00](#), [Gey95](#), [HB15](#), [HM15](#), [Len84](#), [Mai69](#), [McC89](#), [NZ69](#), [Osl06](#), [Ros99](#)].

Undergraduate, master's, and doctoral theses can also be good examples, but they are typically much longer and can vary greatly in intended audience, mathematical correctness, and quality of writing.

4. LIST OF POSSIBLE TOPICS

Here is a list of several possible topics for an expository paper in Diophantine approximation. You can choose a topic not on this list, so long as it is about Diophantine approximation. These are intended to serve as inspiration; you can write about one of these topics as-is, but some may require a narrower focus with you writing about a specific aspect/theorem/example/application/computational experiment within these topics.

Some specific results and conjectures:

- Baker's theory of linear forms in logarithms
- The Coppersmith method
- The Duffin–Schaeffer theorem
- Littlewood's conjecture
- The lonely runner conjecture
- Mahler's problem for the (middle-third) Cantor set
- Oppenheim's conjecture
- The PSLQ algorithm
- Schmidt's conjecture
- Schmidt's subspace theorem
- Zaremba's conjecture
- Zero–one laws in Diophantine approximation

Some specific objects:

- Cantor winning sets
- The Gauss–Kuzmin distribution
- Heilbronn sets
- Hyperplane absolute winning sets
- The Lagrange and Markov spectra
- Quasicrystals

Some general areas of study:

- Approximation by algebraic numbers of bounded degree
- The relation between continued fractions and geodesic flows

- Diophantine approximation on curves and manifolds
- Continued fractions in function fields
- Diophantine approximation on fractal sets
- Inhomogeneous Diophantine approximation
- Multiplicative Diophantine approximation
- Rational approximation with fixed numerators

5. ACKNOWLEDGMENTS

This assignment guide has been loosely adapted from one written by Michel Goemans and Peter Shor for the Fall 2010 course “18.310: Principles of Discrete Applied Mathematics” at MIT.

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