18.784
Communications Workshop: Genre Sources for Research Papers

Spring 2026

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E18-240A

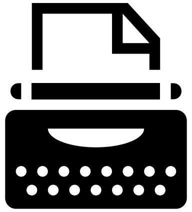
Writing,
Rhetoric,
And
Professional
Communication

Your term paper is an expository paper.

It will synthesize, discuss, and explain *already existing* and *published* results.

As a Writer, You’ll Need to Make Decisions About…

- Your specific focus and purpose for the paper
  - What am I writing about?
  - Why am I writing about that? Meaning, what do I want my readers to gain/learn about/be able to do *after* reading my paper?
- What material to include (or omit)
  - What do I need to cover to help my reader understand the focus and purpose?
  - What can I reasonably assume my reader already knows?
- How to include the material
  - How can I introduce and set up the focus and purpose?
  - How can I organize and structure the material?
  - How can I use language/writing strategies to help my reader understand?



We can learn about strategies for effective mathematical writing by analyzing sample texts

Diophantine Olympics and World Champions: Polynomials and Primes Down Under

Edward B. Burger

1. LET THE GAMES BEGIN: THE OPENING CEREMONIES. For those who think globally, "down under" may provoke thoughts of Australia—the home of the 2000 Olympic Games. For those who think rationally, "down under" may provoke thoughts of denominators of fractions. In this paper, we hope to provoke both.

In basic diophantine approximation, the name of the game is to tackle the following: How close do integer multiples of an irrational number get to integral values? Specifically, if α is an irrational number and the function $\|\cdot\|$ on $\mathbb{R}$ gives the *distance to the nearest integer* (that is, $\|x\| = \min\{|x - m| : m \in \mathbb{Z}\}$), then the game really is a competition among all integers n to minimize the value $\|\alpha n\|$.

Suppose someone serves us an irrational number α. We write $q_1, q_2, q_3, \ldots$ for the (winning) sequence of integers that is able to make it over the following three hurdles:

(i) $0 < q_1 < q_2 < q_3 < \cdots < q_n < \cdots$;
(ii) $\|\alpha q_1\| > \|\alpha q_2\| > \|\alpha q_3\| > \cdots > \|\alpha q_n\| > \cdots$;
(iii) if q is any integer such that $1 \le q < q_n$, $q \neq q_{n-1}$, then $\|\alpha q\| > \|\alpha q_{n-1}\|$.

We now award such a sequence of integers the title: *The world champion approximation sequence for α* or simply say that the q_n's form *the team of world champions for α*. Let us momentarily suppress the natural desire to ask the obvious two questions:

- Do world champions exist for each irrational α?
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and thus we see the q_n "down under" in the fraction p_n/q_n. Suppose that $1 \le q < q_n$ and $\|\alpha q\| = |\alpha q - p|$. Then properties (ii) and (iii) ensure that

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A simple but useful result is

THEOREM I. *Let θ and $Q > 1$ be real. Then there is an integer q such that*

$$0 < q < Q, \quad \|q\theta\| \leqslant Q^{-1}.$$

Note 1. Here θ is not necessarily irrational. The theorem thus gives information on the approximation of rational numbers by rational numbers with smaller denominators.

Note 2. If Q is an integer and $\theta = Q^{-1}$ then $\|q\theta\| \geqslant Q^{-1}$ for $0 < q < Q$. Hence the $\leqslant$ in the theorem cannot be improved to $<$.

First proof. (Dirichlet.) Suppose first that Q is an integer. Consider the distribution of the $Q + 1$ numbers†

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which all satisfy $0 \leqslant x \leqslant 1$, into the Q subintervals

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where $r_2 < r_1$ without loss of generality. Then $q = r_1 - r_2$ does what is required.

Today’s activity asks us to analyze features of the introduction, organization, and language

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These samples were intentionally selected to show two vastly different approaches to writing

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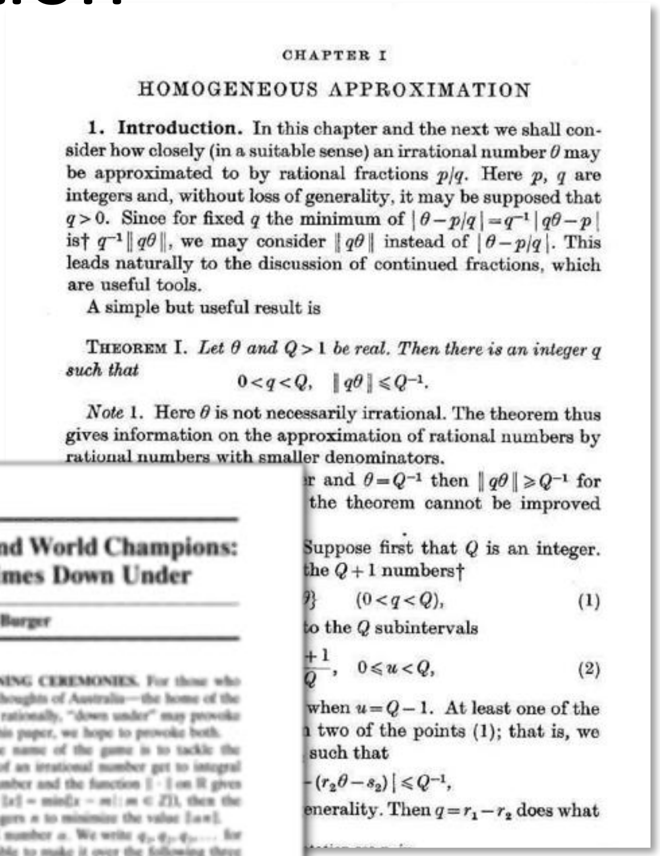
where $r_2 < r_1$ without loss of generality. Then $q = r_1 - r_2$ does what is required.

Consider the following for each text's **introduction**

- What is the paper about? Meaning, what is the mathematical object that is the focus of the paper?
- Where does the paper establish is focus? How?
- What are the main results of the paper?
- Where are they identified? How?
- Consider both texts: What similarities exist across the introductions? What differences? What do these similarities/differences suggest about introductions in math writing?



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- Do world champions exist for each irrational α?
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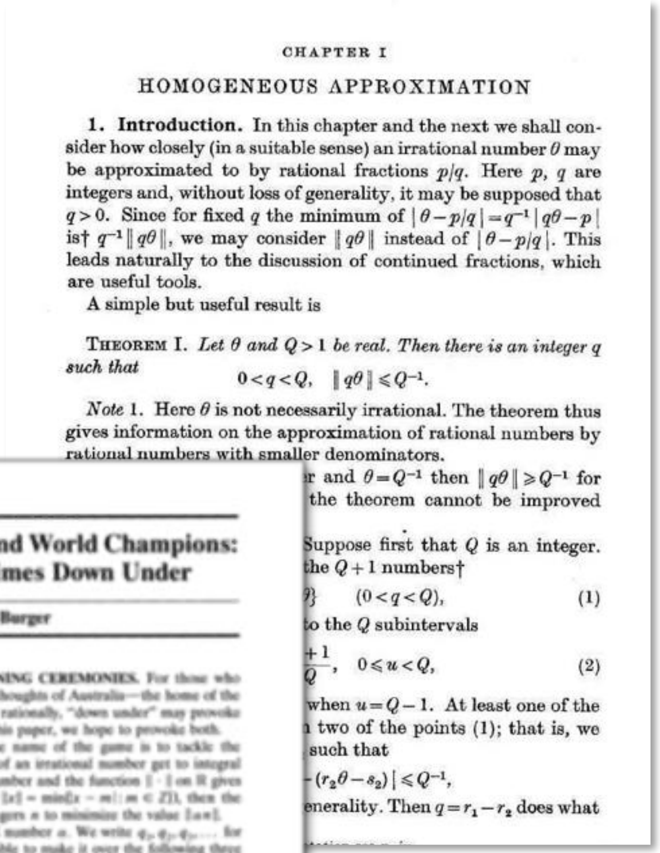
Consider each text's **organization & structure**

- Broadly, how is the paper organized? What topics are discussed, and in what order?
- Why? What might be the rationale for that specific organization/structure?
- How does the paper communicate its organization & structure to the readers? Identify specific examples.
- What features make the text easier to navigate and/or find information?
- Across the two examples, what similarities in organization are there? What differences? What do these similarities/differences suggest about organization in math writing?



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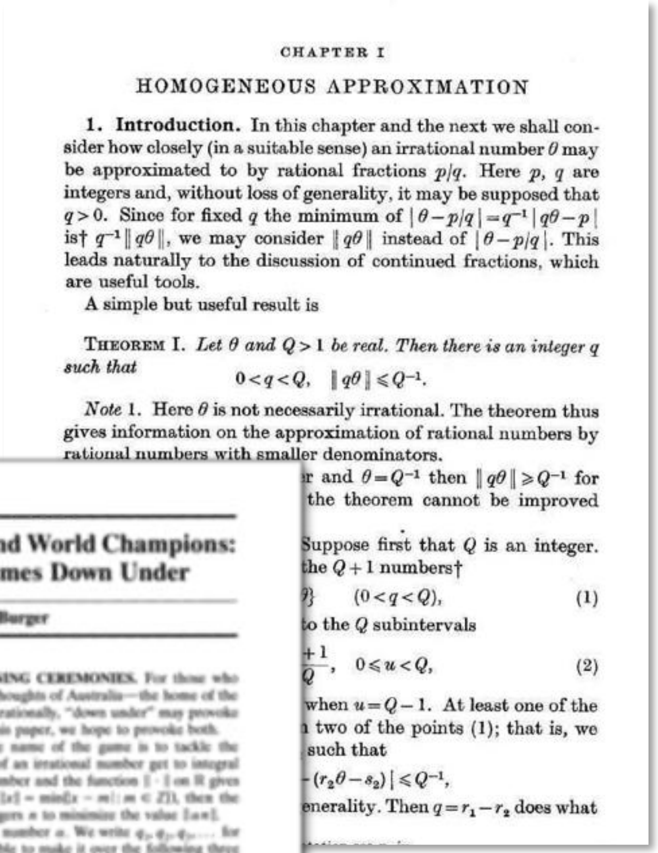
Consider each text’s use of **language**

- What kind of language is used throughout? Consider tone, word choice, formality, etc.
- Identify specific examples of text that **helps** you, as a reader, understand what is happening. Where is that text located? Why is it helpful? What is unique/important about that example’s helpfulness?
- Across the two examples, what similarities in language are there? What differences? What do these similarities/differences suggest about math writing?



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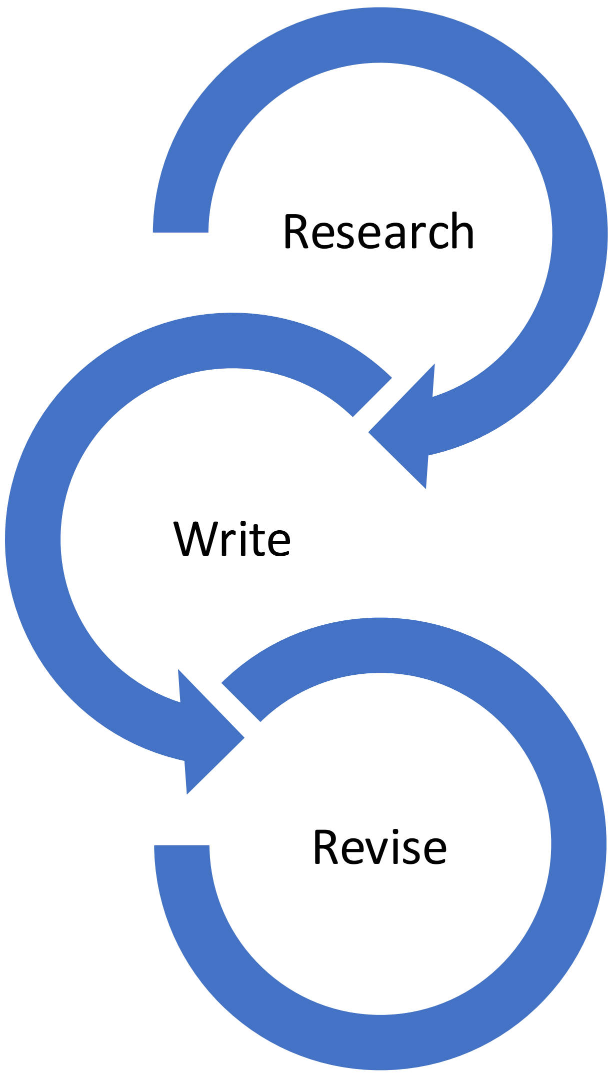
Writing Your Expository Paper



Writing & research are recursive – you will return to and repeat several actions throughout a project!

Writing your first draft…

- Study example texts
  - What are strategies for effective mathematical writing?
  - How can I also use those strategies in my paper?
- Brainstorm your paper
  - Reflect on your research. What is my focus? My purpose?
  - What information will I need to include to do so?
  - How will I include it? (consider intro set up, organization, language)
- Begin writing! Some options…
  - Start at the intro & write your way through the paper, figuring out your focus as you go
  - Start writing the parts you feel most excited about, then fill in other sections afterwards
  - Create an outline, then begin writing
  - **Write in the way that enables you to write – every writer has their own unique writing process ☺**

Remember to cite your sources! Some resources…

In 2011, this paper received the David P. Robbins Prize, one of the MAA's Writing Awards. The annotations presented here provide tips to students for how to write a mathematics paper. Annotations by S. Ruff.

Maximum Overhang

Mike Paterson, Yuval Peres, Mikkel Thorup, Peter Winkler, and Uri Zwick

An article's introduction should 1) indicate the article's main result(s) 2) indicate why the results are important--this is usually accomplished by summarizing how the results further research within the field 3) be worded to be understood by the target audience while remaining relatively nontechnical 4) preview the paper's structure. The first and third goals often conflict with each other.

1. INTRODUCTION. How far can a stack of n identical blocks be made to hang over the edge of a table? The question has a long history and the answer was widely believed to be of order $\log n$. Recently, Paterson and Zwick constructed n-block stacks with overhangs of order $n^{1/3}$, exponentially better than previously thought possible. We show here that order $n^{1/3}$ is indeed best possible, resolving the long-standing overhang problem up to a constant factor.

This paper does a very nice job of addressing goals 1-3 within the first paragraph, so readers immediately know the focus and relevance of the paper.

This problem appears in physics and engineering textbooks from as early as the mid-19th century (see, e.g., [**15**], [**20**], [**13**]). The problem was apparently first brought to the attention of the mathematical community in 1923 when J. G. Coffin [**2**] posed it in the "Problems and Solutions" section of this MONTHLY; no solution was presented there. The problem recurred from time to time over subsequent years, e.g., [**17, 18, 19, 12, 6, 5, 7, 8, 1, 4, 9, 10**], achieving much added notoriety from its appearance in 1964 in Martin Gardner's "Mathematical Games" column of *Scientific American* [**7**] and in [**8**, Limits of Infinite Series, p. 167].

literature review

To indicate how a paper's results further research within the field, the introduction usually includes a literature review. A well-written review gives readers confidence that the authors are familiar with the relevant literature. Furthermore, because the state of the field constantly evolves, the literature reviews in introductions often provide the primary means for those new to a field (e.g., graduate students) to get to know the field. But the

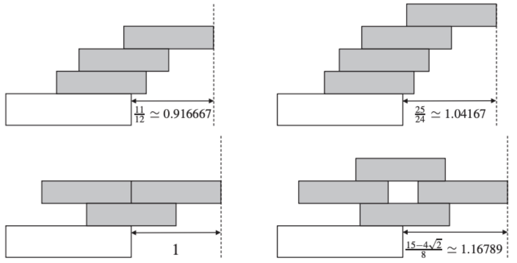


Figure 1. Optimal stacks with 3 and 4 blocks, compared to the corresponding harmonic stacks. The 4-block solution is from [**1**]. Like the harmonic stacks it can be made stable by minute displacements.

Annotated journal article "Maximum Overhang"

Acknowledging Sources in Mathematics Papers

Unless you cite a source, you imply that all wording and results in your paper are yours. If your paper includes information taken from elsewhere, then you must acknowledge the source of the information. This document explains how to appropriately use and acknowledge sources in a mathematics paper.

Avoid using wording from sources In your paper, the writing should be your own, so try to avoid using wording from your sources. If you are writing a paper that summarizes the work of others, then use your own words to synthesize information from a combination of sources: don't rely on one source. To avoid copying wording, understand what your sources are saying, and then put them away before writing. If you must use notes, take sparse notes and avoid using wording from your sources in your notes. It's fine if your resulting explanations are not as elegant as the explanations in your sources, as long as your explanations are correct.

Occasionally, to be accurate and precise, you must present a theorem or definition using the wording that appears in your source. You do *not* need to place the theorem or definition in quotation marks. Instead, clearly state that the theorem or definition is taken from your source, and include a citation.

Even if you use no wording from a source, you must still cite the source if you present information from it.

Examples of how to cite information or wording The introduction should end with a summary of the contents of the paper; in that summary (also called a "road map," you can indicate what information comes from your sources and what information is original. Some examples follow. All examples are taken from the 2002 *Undergraduate Journal of Mathematics*, with minor editorial modifications.

From an introduction:

> The result treated in this paper was proved first by Kakutani in [4]. Intuitively it states this: let *f* be a continuous function defined from a convex subset of an Euclidean space to a set of convex subsets of that set; then *f* has at least one fixed point. The sense in which *f* is continuous and has a fixed point is formalized in Section 2…
>
> —C. Chiscanu, *Fixed-Point Theorems*

From the same paper's road map at the end of the introduction:

> …Section 2 gives the basic definitions, and reviews a proof of Kakutani's Theorem suggested by Hongu He (private communication, June 1998). Section 3 proves a generalization, due to the author, in which the original continuity assumption is relaxed.
>
> —C. Chiscanu, *Fixed-Point Theorems*

From another paper's road map at the end of the introduction:

> In Section 2, we discuss the neoclassical, monopsony and revisionists' models characterizing economists' views on the minimum-wage controversy. In Section 3, using these models, we analyze the debate spurred by Card and Krueger [1]. In Section 4, we assess Card and Krueger's assertions. Finally, in Section 5, in a replication and extension study, we strengthen our conclusions with empirical evidence.
>
> —M. Fernandez, *The Minimum-Wage Controversy*

Acknowledge each source again at the point at which it is used within the body of the paper. If a section relies heavily on a particular source, you may acknowledge that source once at the beginning of the section. Indicate how closely your explanation follows the explanation in your source by using such words as "with slight modifications," "follows," and "based on."

> Now that we have in intuitive picture, we present a formal statement and proof of Menger's theorem. This proof comes from West [2, pp. 149–51]. There exist more than fifteen different proofs of Menger's theorem. West's proof, although not the shortest, is more intuitive than others. We follow it with some slight modifications.
>
> —R. Miller, *Introduction to Network Theory*

Citations should include page numbers, so readers can easily go to your sources to find more information. If you must cite a web page, include the date on which you visited the page, because web pages change.

Common Knowledge It isn't necessary to cite information that is common knowledge. If you aren't sure whether a topic is "common knowledge" for your audience, one option may be to write, "[topic] is presented in most introductory [name of field] texts; for example, see [citation]." Such a note helps readers who want to know more about the topic.

S. Ruff 2/19/08 Based on a handout developed in collaboration with S. Kleiman for 18.096

Handout "Acknowledging Sources in Expository Papers"