Calculus I (UN1101) Practice Midterm #1

Instructor: Robin Zhang

Student Name (UNI): _____

Instructions:

This exam contains **5 pages** (including this cover page) and **5 questions**. The total number of possible points is **34 points**. You will have **65 minutes** to complete this exam.

- **Print your name and UNI** in the space above.
- **Answer the questions in the space provided** on the question sheets. You may use extra paper.
- **Clearly identify and simplify your answers**. You will not receive full credit if there are multiple apparent answers, even if one of them is correct.
- **Write legibly and show your work**. You may receive partial credit for intermediate steps. Correct answers without any reasoning or work will not receive full credit.
- **No calculators, computational devices, or consulting other people** during the duration of this exam. Any cheating will result in an automatic failing grade in the course and potential administrative action.
- **You may consult your notes and textbook** for this exam. This does not include WebAssign, Courseworks, or other online resources.

Do not write in the table to the right.

Question	Points	Score
1	8	
2	12	
3	7	
4	4	
5	3	
Total:	34	

1. Consider the functions

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x \leq -1 \\ x + 1 & \text{if } x > -1 \end{cases}$$
$$g(x) = \cos(x)$$

(a) (2 points) State the domain and range of the function $f(x)$.

(b) (3 points) Sketch the graph of the function $f(x)$.

(c) (3 points) Find the values:

(i) $f(-1)$

(ii) $f(0)$

(iii) $(f \circ g)(0)$

2. Find the limit if it exists. If the limit does not exist, explain why.

(a) (3 points)

$$\lim_{x\to\infty} \frac{x+1}{x}$$

(b) (3 points)

$$\lim_{x\to 1} \frac{x-1}{x^2-1}$$

(c) (3 points)

$$\lim_{x\to 2} 3^{\frac{1}{x-2}}$$

(d) (3 points)

$$\lim_{x\to -1} \frac{4x^2+7x}{x^3+1}$$

3. Consider the function

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & x > 0 \\ x & x \leq 0 \end{cases}$$

(a) (3 points) Is f continuous at $x = 0$? Explain why or why not.

(b) (4 points) Show that f is or is not differentiable at $x = 0$ (and compute $f'(0)$ if it is differentiable) using the definition of the derivative.

4. (4 points) Sketch the graph of an example of a function that satisfies all of the given conditions:

   - $\lim_{x \to -\infty} f(x) = -\infty$
   - $\lim_{x \to 0} f(x) = 0$
   - f(0) = 1
   - $\lim_{x \to 2^-} = \infty$
   - $\lim_{x \to 2^+} = 2$
   - $\lim_{x \to \infty} f(x) = 2$

5. (3 points) What does it mean for a function $f(x)$ to be differentiable at a?
   To receive full credit, provide the definition using limits.