Calculus I (UN1101) Practice Final B

Instructor: Robin Zhang

Student Name (UNI): _____

Instructions:

This exam contains **8 pages** (including this cover page) and **7 questions**. The total number of possible points is **80 points**. You will have **150 minutes** to complete this exam.

- **Print your name and UNI** in the space above.
- **Answer the questions in the space provided** on the question sheets. You may use extra paper.
- **Clearly identify and simplify your answers**. You will not receive full credit if there are multiple apparent answers, even if one of them is correct.
- **Write legibly and show your work**. You may receive partial credit for intermediate steps. Correct answers without any reasoning or work will not receive full credit.
- **No calculators, computational devices, or consulting other people** during the duration of this exam. Any cheating will result in an automatic failing grade in the course and potential administrative action.
- **You may consult your notes and textbook** for this exam. This does not include WebAssign, Courseworks, or other online resources.

Do not write in the table to the right.

Question	Points	Score
1	12	
2	12	
3	10	
4	16	
5	15	
6	6	
7	9	
Total:	80	

1. (12 points) Sketch the graph of a function that is defined on all of the real numbers and satisfies all of the given conditions:

   - f has a horizontal asymptote of $y = 3$
   - $\lim_{x \to -1} f(x) = \infty$
   - f(0) = 0
   - $\lim_{x \to 2^+} = 2$
   - f is increasing on $[2, 3]$
   - $\lim_{x \to \infty} f(x) = \infty$

2. Find the limit if it exists. If the limit does not exist, explain why.

   (a) (4 points)

$$\lim_{x \to 1} \frac{|x-1|}{3}$$

   (b) (4 points)

$$\lim_{x \to -\sqrt{2}} (f \circ g)(x),$$

   where $f(x) = x^2$ and $g(x) = x + 1$

   (c) (4 points)

$$\lim_{x \to \infty} \frac{x^2 + 1}{\ln(2x)}$$

3. Consider the curve given by the equation

$$2y + x^2 y = 2\ln(x)$$

(a) (6 points) Find $\frac{dy}{dx}$ in terms of x and y.

(b) (4 points) Find the equation of the tangent line to the curve at $(1, 0)$.

4. Find the derivative or integral. If the derivative or integral does not exist, explain why.

(a) (4 points)

$$\frac{d}{dx}\left(3x^2\ln(x)\right)$$

(b) (4 points)

$$\frac{d}{dt}\left(\cos\left(e^t\right)+2t\right)$$

(c) (4 points)

$$\int_{-1}^{1}\left(\frac{x^6}{2}-1\right)dx$$

(d) (4 points)

$$\int\left(2x^2+\cos(x)\right)dx$$

5. Consider the function

$$f(x) = \cos(x)^x.$$

(a) (6 points) Find $f'(0)$.

(b) (6 points) Approximate $\cos\left(\frac{\pi}{2}\right)^{\frac{\pi}{2}}$ using the linearization of $\cos(x)^x$ at $x = 0$.

(c) (3 points) Is the linear approximation of $\cos\left(\frac{\pi}{2}\right)^{\frac{\pi}{2}}$ in part (b) and overestimate, underestimate, or neither?

6. (6 points) An asteroid is directly approaching the Earth with velocity (in thousands of miles per hour) at time t (in hours) given by

$$v_{\text{ast}}(t) = 10 + 2te^{(3t^2)}.$$

How far does the asteroid travel in the first 10 hours after time $t = 0$?

7. Consider the functions

$$f(x) = x^2 + 1$$
$$g(x) = 4x + 1$$

(a) (3 points) Find the two points where the curves $y = f(x)$ and $y = g(x)$ intersect.

(b) (6 points) Find the area of the region bounded by $y = f(x)$ and $y = g(x)$.
Hint: $4x + 1 \geq x^2 + 1$ *on the interval* $[0, 4]$.