

Student Name (UNI): _____

Instructions:

This exam contains **10 pages** (including this cover page) and **6 questions**. The total number of possible points is **80 points**. You will have **150 minutes** to complete this exam.

- **Print your name and UNI** in the space above.
- **Answer the questions in the space provided** on the question sheets. You may use extra paper.
- **Clearly identify and simplify your answers.** You will not receive full credit if there are multiple apparent answers, even if one of them is correct.
- **Write legibly and show your work.** You may receive partial credit for intermediate steps. Correct answers without any reasoning or work will not receive full credit.
- **No calculators, computational devices, or consulting other people** during the duration of this exam. Any cheating will result in an automatic failing grade in the course and potential administrative action.
- **You may consult your notes and textbook** for this exam. This does not include WebAssign, Courseworks, or other online resources.

Question	Points	Score
1	12	
2	16	
3	21	
4	9	
5	16	
6	6	
Total:	80	

Do not write in the table to the right.

1. Consider the function

$$f(x) = \begin{cases} \sin(x) & \text{if } x > \pi \\ x - \pi & \text{if } x \leq \pi \end{cases}$$

- (a) (4 points) Identify the real numbers at which $f(x)$ is discontinuous.

Hint: You should justify why $f(x)$ is discontinuous at certain values of x and why $f(x)$ is continuous everywhere else.

- (b) (4 points) Identify the horizontal and vertical asymptotes of $f(x)$.

- (c) (4 points) What does the Mean Value Theorem say about $f(x)$ on the interval $[2\pi, 3\pi]$?

2. Find the limit if it exists. If the limit does not exist, explain why.

(a) (4 points)

$$\lim_{x \rightarrow 1} \frac{x^2 + 1}{x - 1}$$

(b) (4 points)

$$\lim_{x \rightarrow \infty} \frac{e^{-x}}{e^x + 1}$$

(c) (4 points)

$$\lim_{x \rightarrow \infty} \frac{2x^2 - 2}{-2x + \sin(x)}$$

(d) (4 points)

$$\lim_{x \rightarrow 0} x^2 \cos\left(\frac{1}{x}\right)$$

3. Consider the function

$$f(x) = 2x^3 + 9x^2 + 12x + 1.$$

(a) (3 points) State the domain and range of the function $f(x)$.

(b) (4 points) Find $f'(x)$ and $f''(x)$.

(c) (5 points) Find the local extrema of $f(x)$.

(d) (4 points) Find all of the values of x where $f(x)$ has an inflection point.

- (e) (5 points) Sketch a graph of $y = f(x)$ and label its extrema, inflection points, and asymptotes.

4. The spread of a rumor over time within a town with a population of 12000 people can be modeled by a logistic function

$$R(t) = \frac{12000}{1 + e^{-t}}.$$

The logistic function $R(t)$ gives the number of people at time t who have heard the rumor.

- (a) (6 points) Find the linearization of $R(t)$ at $t = 0$.

- (b) (3 points) Use linear approximation at $t = 0$ to estimate the number of people in the town who have heard the rumor at time $t = 1$.

5. Consider the curve $y = x^3 + x$.

- (a) (5 points) Approximate the area under the curve between $x = 0$ and $x = 4$ by a Riemann sum of four rectangles using right endpoints.

- (b) (4 points) Express the area under the curve as the limit of a Riemann sum.

You do not need to evaluate the area for this part. You can leave your answer as a limit.

- (c) (5 points) Find the exact area under the curve between $x = 0$ and $x = 4$.

Hint: you do not have to use the limit from part (b); you find the area using any method covered in class.

- (d) (2 points) Is the Riemann sum approximation of the area under the curve from part (a) an overestimate, an underestimate, or neither?

6. (6 points) Evaluate

$$\int \left(\frac{2}{\cos^2(t)} \sqrt{6 + \frac{\sin(t)}{\cos(t)}} + t \right) dt.$$