

Student Name (UNI): \_\_\_\_\_

**Instructions:**

This exam contains **8 pages** (including this cover page) and **7 questions**. The total number of possible points is **80 points**. You will have **150 minutes** to complete this exam.

- **Print your name and UNI** in the space above.
- **Answer the questions in the space provided** on the question sheets. You may use extra paper.
- **Clearly identify and simplify your answers.** You will not receive full credit if there are multiple apparent answers, even if one of them is correct.
- **Write legibly and show your work.** You may receive partial credit for intermediate steps. Correct answers without any reasoning or work will not receive full credit.
- **No calculators, computational devices, or consulting other people** during the duration of this exam. Any cheating will result in an automatic failing grade in the course and potential administrative action.
- **You may consult your notes and textbook** for this exam. This does not include WebAssign, Courseworks, or other online resources.
- **Upload your exam to Gradescope** via PDF or images at the end of the time allotted. At the end of the exam, you have 15 minutes to upload your exam. Any exams uploaded after the end of the upload period will not be accepted.
- **Remain in the Zoom call with your camera.** If you need to leave your work area for any reason, please inform the instructor beforehand.

Question	Points	Score
1	12	
2	12	
3	10	
4	16	
5	15	
6	6	
7	9	
Total:	80	

Do not write in the table to the right.

1. (12 points) Sketch the graph of a function that is defined on all of the real numbers and satisfies all of the given conditions:
- $f$  has a horizontal asymptote of  $y = 3$
  - $\lim_{x \rightarrow -1} f(x) = \infty$
  - $f(0) = 0$
  - $\lim_{x \rightarrow 2^+} = 2$
  - $f$  is increasing on  $[2, 3]$
  - $\lim_{x \rightarrow \infty} f(x) = \infty$

2. Find the limit if it exists. If the limit does not exist, explain why.

(a) (4 points)

$$\lim_{x \rightarrow 1} \frac{|x - 1|}{3}$$

(b) (4 points)

$$\lim_{x \rightarrow -\sqrt{2}} (f \circ g)(x),$$

where  $f(x) = x^2$  and  $g(x) = x + 1$

(c) (4 points)

$$\lim_{x \rightarrow \infty} \frac{x^2 + 1}{\ln(2x)}$$

3. Consider the curve given by the equation

$$2y + x^2y = 2\ln(x)$$

(a) (6 points) Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

(b) (4 points) Find the equation of the tangent line to the curve at  $(1, 0)$ .

4. Find the derivative or integral. If the derivative or integral does not exist, explain why.

(a) (4 points)

$$\frac{d}{dx} (3x^2 \ln(x))$$

(b) (4 points)

$$\frac{d}{dt} (\cos(e^t) + 2t)$$

(c) (4 points)

$$\int_{-1}^1 \left( \frac{x^6}{2} - 1 \right) dx$$

(d) (4 points)

$$\int (2x^2 + \cos(x)) dx$$

5. Consider the function

$$f(x) = \cos(x)^x.$$

(a) (6 points) Find  $f'(0)$ .

(b) (6 points) Approximate  $\cos\left(\frac{\pi}{2}\right)^{\frac{\pi}{2}}$  using the linearization of  $\cos(x)^x$  at  $x = 0$ .

(c) (3 points) Is the linear approximation of  $\cos\left(\frac{\pi}{2}\right)^{\frac{\pi}{2}}$  in part (b) an overestimate, underestimate, or neither?

6. (6 points) An asteroid is directly approaching the Earth with velocity (in thousands of miles per hour) at time  $t$  (in hours) given by

$$v_{\text{ast}}(t) = 10 + 2te^{(3t^2)}.$$

How far does the asteroid travel in the first 10 hours after time  $t = 0$ ?

7. Consider the functions

$$f(x) = x^2 + 1$$

$$g(x) = 4x + 1$$

(a) (3 points) Find the two points where the curves  $y = f(x)$  and  $y = g(x)$  intersect.

(b) (6 points) Find the area of the region bounded by  $y = f(x)$  and  $y = g(x)$ .

*Hint:  $4x + 1 \geq x^2 + 1$  on the interval  $[0, 4]$ .*