Calculus I (UN1101) Practice Final A

Instructor: Robin Zhang

Student Name (UNI): ____________________

Instructions:

This exam contains **10 pages** (including this cover page) and **6 questions**. The total number of possible points is **80 points**. You will have **150 minutes** to complete this exam.

- **Print your name and UNI** in the space above.
- **Answer the questions in the space provided** on the question sheets. You may use extra paper.
- **Clearly identify and simplify your answers**. You will not receive full credit if there are multiple apparent answers, even if one of them is correct.
- **Write legibly and show your work**. You may receive partial credit for intermediate steps. Correct answers without any reasoning or work will not receive full credit.
- **No calculators, computational devices, or consulting other people** during the duration of this exam. Any cheating will result in an automatic failing grade in the course and potential administrative action.
- **You may consult your notes and textbook** for this exam. This does not include WebAssign, Courseworks, or other online resources.
- **Upload your exam to Gradescope** via PDF or images at the end of the time allotted. At the end of the exam, you have 15 minutes to upload your exam. Any exams uploaded after the end of the upload period will not be accepted.
- **Remain in the Zoom call with your camera.** If you need to leave your work area for any reason, please inform the instructor beforehand.

Do not write in the table to the right.

Question	Points	Score
1	12	
2	16	
3	21	
4	9	
5	16	
6	6	
Total:	80	

1. Consider the function

$$f(x) = \begin{cases} \sin(x) & \text{if } x > \pi \\ x - \pi & \text{if } x \leq \pi \end{cases}$$

(a) (4 points) Identify the real numbers at which $f(x)$ is discontinuous.
Hint: You should justify why $f(x)$ is discontinuous at certain values of x and why $f(x)$ is continuous everywhere else.

(b) (4 points) Identify the horizontal and vertical asymptotes of $f(x)$.

(c) (4 points) What does the Mean Value Theorem say about $f(x)$ on the interval $[2\pi, 3\pi]$?

2. Find the limit if it exists. If the limit does not exist, explain why.

(a) (4 points)

$$\lim_{x \to 1} \frac{x^2 + 1}{x - 1}$$

(b) (4 points)

$$\lim_{x \to \infty} \frac{e^{-x}}{e^x + 1}$$

(c) (4 points)

$$\lim_{x \to \infty} \frac{2x^2 - 2}{-2x + \sin(x)}$$

(d) (4 points)

$$\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right)$$

3. Consider the function

$$f(x) = 2x^3 + 9x^2 + 12x + 1.$$

(a) (3 points) State the domain and range of the function $f(x)$.

(b) (4 points) Find $f'(x)$ and $f''(x)$.

(c) (5 points) Find the local extrema of $f(x)$.

(d) (4 points) Find all of the values of x where $f(x)$ has an inflection point.

(e) (5 points) Sketch a graph of $y = f(x)$ and label its extrema, inflection points, and asymptotes.

4. The spread of a rumor over time within a town with a population of 12000 people can be modeled by a logistic function

$$R(t) = \frac{12000}{1 + e^{-t}}.$$

The logistic function $R(t)$ gives the number of people at time t who have heard the rumor.

(a) (6 points) Find the linearization of $R(t)$ at $t = 0$.

(b) (3 points) Use linear approximation at $t = 0$ to estimate the number of people in the town who have heard the rumor at time $t = 1$.

5. Consider the curve $y = x^3 + x$.

   (a) (5 points) Approximate the area under the curve between $x = 0$ and $x = 4$ by a Riemann sum of four rectangles using right endpoints.

   (b) (4 points) Express the area under the curve as the limit of a Riemann sum.
   You do not need to evaluate the area for this part. You can leave your answer as a limit.

(c) (5 points) Find the exact area under the curve between $x = 0$ and $x = 4$.
Hint: you do not have to use the limit from part (b); you find the area using any method covered in class.

(d) (2 points) Is the Riemann sum approximation of the area under the curve from part (a) an overestimate, an underestimate, or neither?

6. (6 points) Evaluate

$$\int \left(\frac{2}{\cos^2(t)} \sqrt{6 + \frac{\sin(t)}{\cos(t)}} + t \right) dt.$$