# A SUPPLEMENT ON MARKOV CHAIN MONTE CARLO

The handout "Markov chain Monte Carlo" as of today mentions on p. 5, third paragraph, a "Wikipedia article," but in lieu of such an article, here are some remarks.

There are at least two relevant WP articles, "Metropolis–Hastings algorithm" and "Markov Chain Monte Carlo," which itself mentions the Metropolis–Hastings algorithm (to be called MH algorithm here) although without precise definitions.

The distribution of proposed moves from a given $x$, $Q(x, \cdot)$, may be for example the normal $N(x, \sigma^2 I)$. Why might one want to adjust $\sigma$ during the process?

Suppose $\sigma$ were too large, then one might be taking long jumps leading to regions where the target $P(x)$ is small, and so rejecting moves too often and seldom actually moving.

If $\sigma$ were too small, then one might be taking small steps not changing the value of $P(x)$ very much, and the process would be slow in exploring the sample space. For example, in PS8, problem 3, for $P(x)$ having two sharp modes, if $\sigma$ is small it is hard to get away from one mode and move to the other.

So one might experiment with $\sigma$ during early stages, in what has been called a "burn-in" period, aiming at some desired acceptance rate. The article "Metropolis–Hastings algorithm", near the end of the text, says the ideal acceptance rate for a one-dimensional Gaussian distribution is about 50%, decreasing to about 23% for an $N$-dimensional target distribution. This is actually unchanged since October 2012. I don't know where the rate 0.6 mentioned in PS8, problem 3 came from, but rather than changing the problem, it will be left as is; this is in one dimension, and 60% is not all that different from 50%.

"Burn-in period" can also make sense if the proposal distribution is unchanged, as in a relatively short time the distribution of the Markov chain's position will not yet come close to the target distribution. In the WP article "Metropolis–Hastings algorithm" p. 4, it's suggested that the first 1,000 or so steps should be thrown away.

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