Propagation of singularities and the structure of the scattering matrices in many-body scattering

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In this talk I will extend the familiar example that singularities of solutions of the wave equation propagate along (generalized) broken bicharcteristics, to many-body scattering. Such a description connects geometric optics, where light is modelled by little billiard balls, moving in straight lines and reflecting from surfaces according to Snell's law, and wave phenomena. In this talk I will discuss an analogous connection between quantum and classical many-body systems.

Let H be a many-body Hamiltonian, so $H = \Delta + V$, $V = \sum_a V_a$, where the two-body interactions V_a are real-valued polyhomogeneous symbols of order -1 (e.g. Coulomb-type with the singularity at the origin removed). I will discuss the propagation of singularities of generalized eigenfunctions of H, where 'singularities' are understood as a description of the lack of decay at infinity. In particular, I will explain the propagation results in three-body scattering, and also in many-body scattering under the additional assumption that no subsystem has a bound state. These results provide a connection between quantum mechanics and classical mechanics which is very similar to the one between wave propagation and geometric optics discussed above. They also prove that the wave front relation of the free-to-free S-matrix (which is all of the S-matrix if no subsystem has a bound state) is given by the broken geodesic flow, broken at the 'singular directions' corresponding to the collision planes, on $\mathbb{S}^{n-1}$ at time π. In many-body scattering with at least four particles, if some subsystems have bound states, one would have to combine the behavior of bound states with the classical dynamics to understand the propagation of singularities.

These results have natural geometric generalizations on asymptotically Euclidean spaces X with Hamiltonians that are singular at certain submanifolds of the geodesic compactification of X.