# Short time behavior of logarithmic derivatives of the heat kernel

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Abstract: Let $M$ be a compact connected Riemannian manifold and $\Delta$ be the (Levi-Civita) Laplacian. Let $p_t(x, y)$ be the fundamental solution to the Cauchy initial value problem for the heat equation $\frac{\partial u}{\partial t} = \Delta u$. We derive the upper bounds which say that the $n$-th covariant logarithmic derivative of $p_t(\cdot, y)$ at $x$ behaves like a constant times $\left(\frac{1}{t} + \frac{dist(x,y)^2}{T^2}\right)^{\frac{n}{2}}$, for $t \in (0, 1]$. We then study the asymptotic behavior of logarithmic derivatives. In particular, we show that this bound can be improved for compact sets in the complement of the cut locus of $y$, and that a dramatic change in behavior occurs for $x$ at the cut locus of $y$.