9. Eta.

First generalize to the geometric case

Bundles suppressed

$$\Psi^{-\infty}_{sm}(X) \longrightarrow \Psi^{\mathbb{Z}}_{sm}(X) \longrightarrow A^{\mathbb{Z}}_{sm}(X)$$

$$A^{\mathbb{Z}}_{sm}(X) = C^\infty(S(\mathbb{R}\times T^*X))\,[\Gamma\rho\rho^{-1}]$$

pseudo differential operators with "symbolic" parameters

sphere bundle of [cotangent bundle with extra trivial dim

$$G^{-\infty}_{sm}(X) \longrightarrow G^{\mathbb{Z}}_{sm}(X) \longrightarrow E^{\mathbb{Z}}_{sm}(X)$$

$$c_0 \downarrow \qquad\qquad \eta \downarrow$$

$$\mathbb{Z} \qquad\qquad \mathbb{C}$$

$$c_0 = \frac{1}{2\pi i}\int \mathrm{Tr}\left(A^{-1}\frac{dA}{ds}\right) = \frac{1}{2\pi i}\,\mathrm{Tr}_I\left(A^{-1}\frac{d}{ds}A\right)$$

$$\eta = \overline{\mathrm{Tr}}_I\left(A^{-1}\frac{dA}{ds}\right)$$

$$\tau = e^{2\pi i \eta} \qquad \text{'determinant'}$$

$$\frac{1}{2\pi i}\,d\eta = \frac{1}{2\pi i}\,\mathrm{Tr}_R\left(A^{-1}\,dA\right)$$

1st odd chern class

$$= \frac{1}{2\pi i}\,d\log\tau$$