8. Bismut-Freed-Quillen

$P: Y \hookrightarrow \Psi^{\mathbb{Z}}(X)$ smooth <u>elliptic</u>

$$\Rightarrow \quad Q_y \cdot P_y = Id - B_y$$
$$P_y \cdot Q_y = Id - A_y$$

can be chosen smooth

$$Y \longrightarrow Id + E_y(s) \in \Psi^{-\infty}_{sm}(X)$$

$\Rightarrow \mathrm{Ind}(P) \in K^{-1}(Y)$ "<u>Virtual</u> index bundle"

$c_0^* \mathrm{Ind}\, P \in \mathbb{Z}$ index $(=0)$

$c_2 = c_1^* \mathrm{Ind}\, P \in H^2(Y)$

BFQ: $c_1^* \mathrm{Ind}\, P = \frac{1}{2\pi i}\omega$

is the curvature of the determinant bundle

Pf: Almost by definition.