5. Index (Trivial unless boundary)

$$\Psi^{-\infty}(X) \to \Psi^{\mathbb{Z}}(X) \xrightarrow{\sigma} A^{\mathbb{Z}}(X)$$

$$\downarrow P \qquad\qquad C^\infty(S^*X)[[\rho]]$$

Q: Is P invertible?

Necessary $\sigma(P)$ is invertible $\sigma(P) \sim \sum_{j=m}^{\infty} a_j \rho^{k-j}$

$\Leftrightarrow a_0$ is invertible "ellipticity"

$\Leftrightarrow \exists Q$ s.t.

$$PQ = Id - A \qquad A, B \in \Psi^{-\infty}(X)$$
$$QP = Id - B$$

A, B can be made self-adjoint projections

$$\mathrm{Ind}(P) = \mathrm{rank}(B) - \mathrm{rank}(A) \in \mathbb{Z}$$

obstructed invertibility.

Consider $E(s) = f^+(s) B - f^-(s) A$

Assume $A \perp B$. $w(1 + f^+(s)) = -w(1 + f^-(s)) = 1$