1. Pass from 'trivial' case of $\mathbb{R}^n$ to a compact manifold X, initially without boundary

Smoothing operators are the 'same'

$\Psi^{-\infty}(X) = C^\infty(X^2)$ with non-commutative product

$$A \cdot B(x,y) = \int A(x,z) B(z,y) \, d\mu$$

Invariant choice of smooth measure

– I will suppress bundles for notational simplicity!

$$HH_k(\Psi^{-\infty}(X)) = \begin{cases} \mathbb{C} & k=0 \\ \{0\} & k>0 \end{cases}$$

Unique trace:

$$Tr(A) = \int_X A(x,x) \, d\mu.$$