7.

$$\begin{array}{ccccc} \mathbb{Z} & & \mathbb{C} & & \\ c_0 \uparrow & & \eta \uparrow & & \\ G^{-\infty}_{\text{hilb}}(n) & \longleftrightarrow & G^{0}_{\text{hilb}}(n) & \longrightarrow & A^{0}_{\text{hilb}}(n) \end{array}$$

$$c_0(P) = \frac{1}{2\pi i} Tr_I \left(P^{-1} \frac{1}{dt} P \right) \qquad P = \mathbb{I} + A$$

$$\eta(P) = \frac{1}{2\pi i} \overline{Tr}_I \left(P^{-1} \frac{1}{dt} P \right)$$

$\overline{Tr}_I$ is a trace

$$\Rightarrow \quad \eta(PQ) = \eta(P) + \eta(Q)$$

$c_0(P)$ is locally constant

$$\frac{1}{ds}\left(P_s^{-1} \frac{1}{dt} P_s\right) = -P_s^{-1} \frac{1}{ds} P_s \cdot P_s^{-1} \cdot \frac{1}{dt} P_s + P_s^{-1} \frac{1}{ds} \frac{1}{dt} P_s$$

$$= \frac{1}{dt}\left(P_s^{-1} \frac{1}{ds} P_s\right) + \left[P_s^{-1} \frac{1}{dt} P_s, P_s^{-1} \frac{1}{ds} P_s\right]$$

$$\Rightarrow \quad \frac{1}{ds} \eta(P_s) = Tr_R \left(P_s^{-1} \frac{1}{ds} P_s \right)$$