3. $\Psi^{-\infty}_{iso}(n)$ is an infinite matrix algebra.

– Analytically

(Take $n=1$). $H = -\frac{d^2}{dx^2} + x^2$

has a complete set of eigenvectors

$$H u_k = (k+1) u_k \qquad k \in \{0, 1, \dots\}.$$

$$f \in \mathcal{S}(\mathbb{R}) \iff f(x) = \sum_{k=0}^{\infty} f_k \, u_k(x)$$

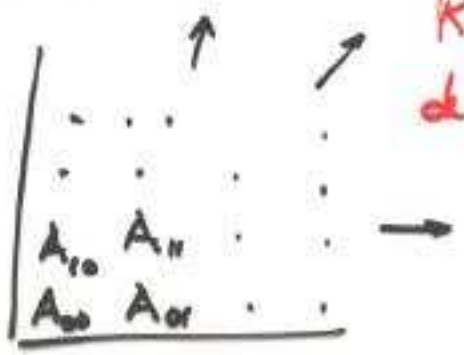
$$\sup_k |k^N f_k| < \infty \quad \forall N$$

$$Af = \sum_{k,j} A_{kj} f_j \, u_k(x)$$

$\Psi^{-\infty}_{iso}(1) \ni A \longleftrightarrow$

$$\begin{array}{|cccc} & & \uparrow & \nearrow \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ A_{10} & A_{11} & \cdot & \cdot \\ A_{00} & A_{01} & \cdot & \cdot \\ \hline \end{array} \longrightarrow$$

Rapidly decaying



$M(N) \hookrightarrow \Psi^{-\infty}_{iso}$

subalgebra

$$\begin{array}{c|cc} & 0 & 0 \\ \hline N & \times & 0 \\ \hline & N & \end{array}$$