2. Smoothing operators

$A \in \mathcal{S}(\mathbb{R}^{2n})$ defines an operator

(= continuous linear map)

$$A : \mathcal{S}'(\mathbb{R}^n) \longrightarrow \mathcal{S}(\mathbb{R}^n)$$

$$Af(x) = \int_{\mathbb{R}^n} A(x,y)\, f(y)\, dy$$

and <u>conversely</u>.

These form an algebra of operators on $\mathcal{S}(\mathbb{R}^n)$:

$$A \circ B(x,y) = \int_{\mathbb{R}^n} A(x,z)\, B(z,y)\, dz.$$

Let $\Psi_{\mathrm{lso}}^{-\infty}(n)$ denote this non-commutative algebra $= \mathcal{S}(\mathbb{R}^{2n})$ with $\circ$.