18.155 LECTURE 12
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RICHARD MELROSE

ABSTRACT. Notes before and then after lecture.

Read: I will add some of this to the notes when I get a chance.

(1) Hilbert-Schmidt operators as a Hilbert space $\|A\| \leq \|A\|_{\mathrm{HS}}$ $\|BA\|_{\mathrm{HS}} \leq \|B\|\|A\|_{\mathrm{HS}}$. Density of finite ranke operators.

(2) Trace class operators, the trace, $\|T\| \leq \|T\|_{\mathrm{Tr}}$, $\|BT\|_{\mathrm{Tr}} \leq \|B\|\|T\|_{\mathrm{Tr}}$, density, $\mathrm{Tr}([B,T]) = 0$. $\mathrm{Tr}(\Pi_N) = N$.

(3) Index of Fredholm operators, left and right parametrices, Calderón's formula.

(4) Fredholm operators form an open set.

(5) Constancy of the index on components.

(6) Spectrum of a bounded operator.

(7) Self-adjoint operators, $\|A\| = \sup_{\|u\|=1} |\langle Au, u \rangle|$.

(8) $P(A)$ for a polynomial P and $\mathrm{Spec}(P(A))$.

(9) The functional calculus

(10) Spectral theorem for compact operators

(11) Spectral projections.

DEPARTMENT OF MATHEMATICS, MASSACHUSETTS INSTITUTE OF TECHNOLOGY
E-mail address: rbm@math.mit.edu