From Rudin, Chapter 1.

Exercise 1 If s and $r \neq 0$ are rational then so are $s + r$, $-r$, $1/r$ and sr (since the rationals form a field). So if r is rational and x is real, then $x + r$ rational implies $(x + r) - r = x$ is rational. An irrational number is just a non-rational real number, so conversely if x is irrational then $x + t$ must be irrational. Similarly if rx is rational then so is $(xr)/r = x$; thus if x is irrational then so is rx.

Exercise 3 [(a)] If $x \neq 0$ then x^{-1} exists and if $xy = xz$ then

$$y = (x^{-1}x)y = x^{-1}(xy) = x^{-1}(xz) = (x^{-1}x)z = z$$

using first (M5) then (M2), (M3), the given condition, (M3) and (M5).

[(b)] Is (a) with $z = 1$.

[(c)] Multiply by x^{-1} so $x^{-1} = x^{-1}(xy) = (x^{-1}x)y = 1y = y$ using associativity and definition of inverse.

[(d)] The identity for $x^{-1} = 1/x$, $x \cdot x^{-1}$ gives by commutativity $x^{-1} \cdot x = 1$ which means $1/(1/x) = x$ by the uniqueness of inverses.

Exercise 5 If A is a set of real numbers which is bounded below then $\inf A$ is by definition a lower bound, i.e. $\inf A \leq a$ for all $a \in A$ and if $\inf A \geq b$ for any other lower bound b. We already know that if it exists it is unique. Now if A is bounded below then

$$-A = \{-x; x \in A\} \tag{1}$$

is bounded above. Indeed if $b \leq x$ for all $x \in A$ then $-b \geq -x$ for all $x \in A$ which means $-b \geq y$ for all $y \in -A$. Now, if $\sup(-A)$ is the least upper bound of $-A$ it follows that $-\sup(-A)$ is a lower bound for A since

$$x \in A \Longrightarrow -x \in -A \Longrightarrow \sup(-A) \geq -x \Longrightarrow -\sup(-A) \leq x.$$

As noted above, if b is any lower bound for A then $-b$ is an upper bound for $-A$ so $-b \geq \sup(-A)$ and $b \leq -\sup(-A)$. This is the definition of $\inf A$ so

$$\inf A = -\sup(-A).$$