18.155 LECTURE 22
30 NOVEMBER, 2017

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Elliptic operators on compact manifolds are Fredholm – combining what we know.

- Local elliptic regularity for systems – the proof recalled. If $P(x, D_x)$ is an $N \times N$ matrix of elliptic operators of order m on an open set $\Omega \subset \mathbb{R}^n$, then
$$u \in \mathcal{C}^{-\infty}(\Omega; \mathbb{C}^N),\ P(x, D)x)u \in H^s_{\text{loc}}(\Omega) \Longrightarrow u \in H^{s+m}_{\text{loc}}(\Omega)$$
and we have estimates! If $\chi_1 \in \mathcal{C}^\infty_{\text{c}}(\Omega)$, $\chi_2 \in \mathcal{C}^\infty_{\text{c}}(\Omega)$, $\text{supp}(1 - chi_2) \cap \text{supp}(\chi_1) = \emptyset$ then for each s and M there are constants C and C' such that
$$\|\chi_1 u\|_{s+m} \le C\|\chi_2 Pu\|_s + C'\|\chi_2 u\|_M. \tag{1}$$
- If V, W are vector bundles over a compact manifold M and $P \in \text{Diff}^m(M; V, W)$ is elliptic then
$$P : H^{s+m}(M; V) \longrightarrow H^s(M; W) \tag{2}$$
is Fredholm for any s with index which does not depend on s.
- Next we want to think about some of the most important examples, the Laplace-Beltrami operator and the closely related Hodge operator (or signature operator).
  Riemann metrics
  Metrics on bundles
  Adjoint operators
  The case of d and δ.
  $\Delta = d\delta + \delta d$.

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