Differential operators on manifolds.
I will go quite quickly through all the ‘preparatory’ material assuming you have had a course on differentiable manifolds. Even so this is more than I will be able to do in one lecture!

- Manifolds, coordinate patches, transition maps.
- $C^\infty(M)$ and smooth maps, Df(M), fibre bundles, vector bundles.
  Suggested reading: Principal bundles, associated bundles.
- Tangent and cotangent bundles, form bundles, density bundle. Metrics.
- Smooth sections, Fréchet topology.
- Differential operators, symbols, ellipticity.
- DeRham differential
- Adjoints,
- Elliptic regularity for matrix-valued operators.
- Distributional and Sobolev spaces of sections.
- Global elliptic regularity
- Elliptic operators over compact manifolds are Fredholm
- The Hodge operator and Hodge theorem