18.155 LECTURE 13 26 OCTOBER, 2017

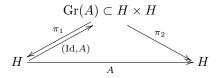
RICHARD MELROSE

No lecture on 24th October. Read: Notes Chapter 2.

- Compact operators as closure of finite rank operators.
- $\overline{\operatorname{Ran} A}^{\perp} = \operatorname{Nul}(A^*).$
- Since there is no lecture Tuesday you can (have) use(d) it to check that Baire's Theorem can be used to show:
 - Uniform Boundedness=Banach-Steinhaus (straightforward)
 - Open Mapping Theorem (trickier)

Closed Graph Theorem (follows easily from $\mathrm{OMT})$ – look at the commutative diagram

(2)



If Gr(A) is closed the continuity of π_1 , and hence A follows from the OMT. Note that if $A : H \longrightarrow H$ is bounded and a bijection, the continuity

- of A^{-1} follows from either the OMT or the CGT.
- Spectrum and resolvent of a bounded operator.
- Spectrum of a compact operator is discrete outside {0}.
- Spectrum of a self-adjoint operator is contained in $\mathbb{R} \subset \mathbb{C}$.
- If $A = A^* \in \mathcal{B}(H)$ then $||A|| = \sup_{||u||=1} |\langle Au, u \rangle|$.
- If $A = A^*$ then $\{\alpha\} \cup \{\beta\} \subset \operatorname{spec}(A) \subset [\alpha, \beta], \ \alpha = \inf_{\|u\|=1} \langle Au, u \rangle, \ \beta = \sup_{\|u\|=1} \langle Au, u \rangle.$
- If $A = A^*$ and p is a polynomial with real coefficients then

$$\|p(A)\| \le \sup_{[\alpha,\beta]} |p(z)|$$

• (Functional Calculus) If $A = A^* \in \mathcal{B}(H)$ there is a continuous linear map of norm one

(3)
$$\mathcal{C}([\alpha,\beta]) \ni f \longmapsto f(A) \in \mathcal{B}(H) \text{ s.t. } f(A)g(A) = (fg)(A).$$

- where $f(A) = A^j$ if $f(x) = x^j$.
- Existence of spectral projections.

DEPARTMENT OF MATHEMATICS, MASSACHUSETTS INSTITUTE OF TECHNOLOGY *E-mail address*: rbm@math.mit.edu