TOPIC LIST FOR STAGE IN SPRING 2013

Each entry below is intended to be covered in a 1-hour seminar talk. Of course this will mean omitting many proofs.

(1) **Hodge theory.** [Voi10a], [Huy12], [Voi02], [Voi03]
- Definition of Hodge structure
- Smooth projective variety X gives polarized Hodge structure on $H^*(X, \mathbb{Z})$ (compatibility of Betti and de Rham cohomologies)
- Lefschetz 1,1 theorem and statement of the Hodge conjecture

(2) **K3 surfaces over an arbitrary field.** [Huy12]
- Definition of algebraic K3 surface
- Properties of cohomology of a K3 surface
- Examples (e.g., Kummer surfaces)

(3) **Clifford algebra and spin group.** [Huy12]
- Clifford algebra of a quadratic space
- The groups Spin and GSpin

(4) **The Kuga-Satake construction.** [Huy12], adapted to use the full Clifford algebra. Perhaps see also [Del72].
- Riemann's equivalence of categories between polarized abelian varieties and polarized Hodge structure of weights $(0, 1)$ and $(1, 0)$
- The Kuga-Satake Hodge structure
- Examples (e.g. Kummer surfaces)

(5) **Moduli of K3 surfaces.** [Huy12]
- Moduli space of K3 surfaces
- Statement of Torelli theorem

(6) **Facts about étale cohomology.** [Del82, Section 1]
- Algebraic nature of ℓ-adic cohomology, Galois action
- Comparison with Betti cohomology
- Cycle class map

(7) **The Tate conjecture.** [Tat94]
- Generalities on the Tate conjecture
- Implications
- Known cases

(8) **The Tate conjecture for abelian varieties.** [Tat66]
- Connection with abelian variety homomorphisms and Tate modules
- Summary of Tate's proof

(9) **Hodge implies absolute Hodge.**
- Deligne's invariant cycles theorem [Voi10b], [Del71]
- Hodge implies absolute Hodge for abelian varieties, K3 surfaces, and products of these [Del82], [Bla94]

Date: January 31, 2013.

(10) **Proof of the Tate conjecture for K3 surfaces over f.g. fields of characteristic 0.**

No good reference?

References

[Bla94] Don Blasius, *A p-adic property of Hodge classes on abelian varieties*, Motives (Seattle, WA, 1991), Proc. Sympos. Pure Math., vol. 55, Amer. Math. Soc., Providence, RI, 1994, pp. 293–308. MR1265557 (95j:14022) ↑9

[Del71] Pierre Deligne, *Théorie de Hodge. II*, Inst. Hautes Études Sci. Publ. Math. **40** (1971), 5–57 (French). MR0498551 (58 #16653a) ↑9

[Del72] ———, *La conjecture de Weil pour les surfaces K*3, Invent. Math. **15** (1972), 206–226 (French). MR0296076 (45 #5137) ↑4

[Del82] ———, *Hodge cycles on abelian varieties*, Hodge cycles, motives, and Shimura varieties, Lecture Notes in Mathematics, vol. 900, Springer-Verlag, Berlin, 1982. Edited and revised by J. S. Milne, available at `http://www.jmilne.org/math/Documents/Deligne82.pdf` . ↑6, 9

[Huy12] Daniel Huybrechts, *Lectures on K3 surfaces*, November 8, 2012. Available at `http://www.math.uni-bonn.de/people/huybrech/K3Global.pdf` . ↑1, 2, 3, 4, 5

[Tat66] John Tate, *Endomorphisms of abelian varieties over finite fields*, Invent. Math. **2** (1966), 134–144. MR0206004 (34 #5829) ↑8

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[Voi03] ———, *Hodge theory and complex algebraic geometry. II*, Cambridge Studies in Advanced Mathematics, vol. 77, Cambridge University Press, Cambridge, 2003 (reprinted in 2007). Translated from the French by Leila Schneps. MR1997577 (2005c:32024b) ↑1

[Voi10a] ———, *On the cohomology of algebraic varieties*, Proceedings of the International Congress of Mathematicians. Volume I, Hindustan Book Agency, New Delhi, 2010, pp. 476–503. MR2827902 ↑1

[Voi10b] ———, *Hodge loci*, 2010. Preprint, `http://www.math.jussieu.fr/~voisin/Articlesweb/hodgeloci.pdf` . ↑9