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A mathematician's life

Alexander Grothendieck, 1928-2014

His childhood

His parents, Sasha and Hanka, wandered the streets of Berlin in the 1920's, dragging their heavy camera from one place to another to snap pictures of the few passersby who were willing to stop and hand over the 30 pfennigs that weren't always easy to come by in those times of radical depression and unemployment. Inveterate anarchists, Alexander Grothendieck's parents rejected all idea of institutionalized or salaried work and believed only in direct contact with people, whether through street photography, singing in smoky bars or selling left-wing newspapers or brochures in the public squares. Alexander grew up in Berlin like a little street urchin, neglected yet beloved, with gaping shoes, ragged clothing and uncut mop of hair, sometimes hungry, and not too familiar with the habits of good breeding. Of this childhood, shattered abruptly in 1933 by the departure of his parents for France to flee Nazism, Alexander kept a memory of total happiness that accompanied him throughout his life. He attributed to these early years the freedom of vision that made him into a more imaginative and productive mathematician than many whom he considered as actually more talented and more brilliant than himself.

His reclusion

In 1991, tired of not succeeding in communicating his ideas to anyone, Grothendieck decided to disappear, leaving his friends and family with no news of his whereabouts. He spent the last 25 years of his life in the small village of Lasserre, in the Pyrenees, in total and voluntary solitude. His neighbors knew that he spent his nights writing, all night, every night. He left many thousands of pages behind, in which he wrote of his life, his vision, the universe and above all of the fundamental question of evil that he spent the last part of his life trying to understand.

Grothendieck in 2013, and his writings

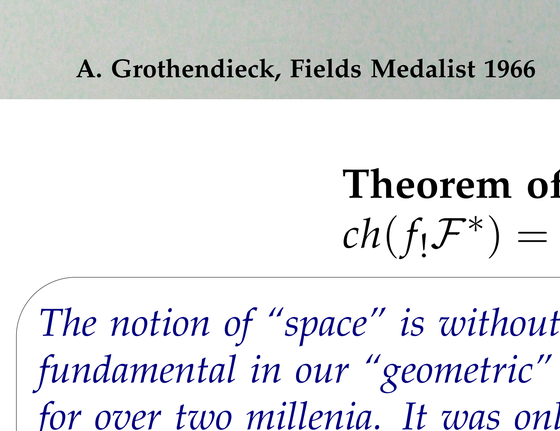
His teaching

Starting in 1974, Grothendieck taught at the university of Montpellier. His ideas were, to say the least, very original. An excerpt from his third-year mathematics course description:

Our main point, then, will be to incite the child that sleeps inside the good student sitting rigidly on his bench (as well as in the professor) to PLAY. But is it really the role of the professor to do this inciting? Isn't it, rather, the role of each one of us to incite all the others, starting with himself? To convince ourselves to do this, in the absence of any particular interest for a given subject that deep down schoolchildren don't give a d... about, don't we need a wave of healthy nausea at the idea of eternally going through the same old mechanical dance, insignificant extras in the infinitely repeated rite of our own castration? Or has the rite already acted on us, successfully castrating in us the free and creative man or woman? Are we really hopelessly reduced to the sad state of Homunculus Studiensis? To our places, then, "Professor" and "Students", to submissively execute the sempiternal ballet!

His activism

Starting in 1970, Grothendieck left his comfortable job at the IHES to devote himself body and soul to ecological questions, militating against nuclear energy, pesticides, technology, weapons, chemical products and everything else that seemed to threaten nature and the place of man at its heart. Leaving behind mathematics and his colleagues, he connected with a new group of people who shared his ideas, and founded a newspaper, *Survivre et Vivre* (*Survive and Live* which reached a circulation of some 10,000 copies. During this period he left his family to live in a commune, and traveled to poor countries, teaching mathematics in Vietnam under American bombs; he even sold his Fields medal and gave the money to Vietnamese refugees, and was taken to trial for the crime of letting illegal immigrants live in his house. After a few years, however, he understood that he was merely preaching to the choir, without having the slightest effect on the wider public. He abandoned the movement and moved to a village in the countryside to continue his intellectual voyage in solitude, devoting himself in large measure to trying to understand the general indifference to what he himself perceived as being of absolute urgency.



Grothendieck in Vietnam, 1967

Grothendieck's last student, Hoang Sinh, met him when he visited Vietnam in 1967. Today she is the president of a private university that she founded in Vietnam.

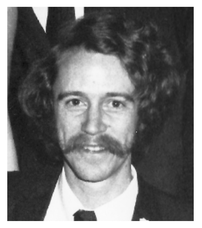
His school

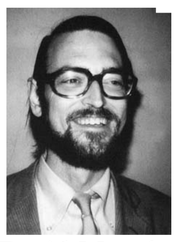
Young Pierre Deligne came to Grothendieck's seminar in 1967. Later, he proved the last of the *Weil conjectures*, thus completing one of Grothendieck's major projects. He was awarded the Fields medal in 1978.

After defending her doctoral thesis under Grothendieck in 1972, Monique Hakim became a professor at the University of Paris and turned to more analytic research topics.

Professor at the University of Paris, Luc Illusie, who was close to Grothendieck for many years, continued to work on Grothendieck-style algebraic geometry throughout his career.

Max Karoubi welcomed Grothendieck in Algeria after the Algerian war, sharing his attitude towards the student revolution of May 1968. He devoted his research career to the development of Grothendieck's K-theory.

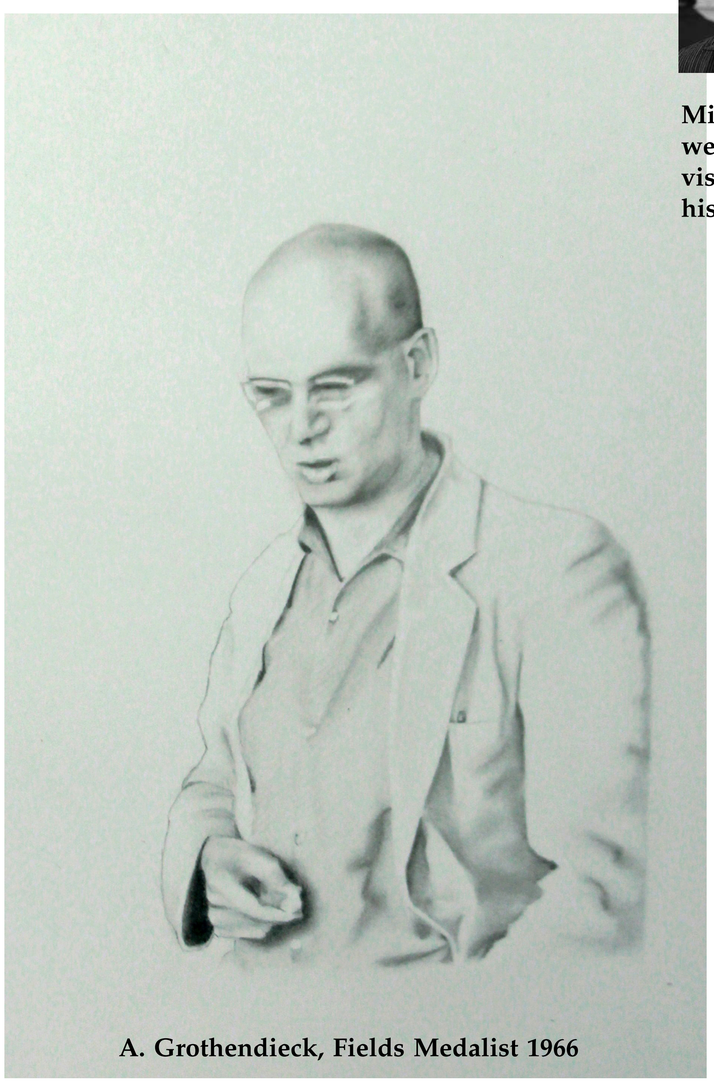




Mike Artin, Steve Kleiman, Robin Hartshorne and David Mumford were part of the group of American mathematicians who regularly visited Paris, invited Grothendieck to the U.S. and greatly furthered his mathematical initiatives.

His research

In 1948, at the age of 20 and with a B.A. in mathematics from Montpellier, Grothendieck went to Paris to start on a research career. Four years later, he defended an impressive doctoral thesis in Nancy. But being stateless, he could not hold a university position in France. For three years he held temporary positions in North and South America. Following this period, he proved a grand generalization of the classical theorem of Riemann-Roch which deepened and clarified the scope of the original theorem. This result projected him to instant stardom and gained him a job in a specially created research institute in Paris, the IHES (Institut des Hautes Études Scientifiques), created specially with him in view. He didn't much like the style of his own proof, however, and refused to publish it. For him, the pursuit of mathematics amounted to a permanent and uncompromising search for absolute purity and he detested anything that looked to him like a gimmick or a trick. The statement of his Riemann-Roch theorem, even if one doesn't understand it, is typical Grothendieck in its concise formulation, its economy and its concision which reflects the beauty of simplicity.



A. Grothendieck, Fields Medalist 1966

Theorem of Riemann-Roch-Grothendieck:

$$ch(f_!\mathcal{F}^*) = f_*\big(ch(\mathcal{F}^*)td(T_f)\big).$$

The notion of "space" is without a doubt one of the oldest in mathematics. It is so fundamental in our "geometric" grasp of the world that it remained more or less tacit for over two millenia. It was only during the past century that this notion has detached itself, little by little, from the tyrannical domination of our immediate perception (of a single "space", the one surrounding us), and of its traditional theorization ("Euclidean"), and acquiring its own autonomy and dynamics. People have invented all sorts of "measuring devices" or "rulers" that can be used, in spite of all obstacles, to attach certain "measurements" to these tentacular "spaces" that seem to want to steal away, like an elusive fog, and avoid all efforts to measure them.

By inventing *schemes* and *toposes*, Grothendieck widened the classical notions of geometric or topological space. According to his vision, it was important to turn one's attention away from the elements of the space, namely its points, to concentrate instead on the characteristics of the space, such as the measures of distance, the curvature or the number of holes, to the point where one should stop considering the space as a set of points to be described, but rather think of it directly as identified with its own description by measurements.