Computing the Multivariate Hypergeometric Function

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Abstract

The *multivariate* hypergeometric function

$${}_{p}F_{q}^{\alpha}(a_{1},\ldots,a_{p};b_{1},\ldots,b_{q};x_{1},\ldots,x_{n}) = \sum_{\kappa} \frac{(a_{1})_{\kappa}\ldots(a_{p})_{\kappa}}{(|\kappa|)!(b_{1})_{\kappa}\ldots(b_{q})_{\kappa}} \cdot C_{\kappa}^{\alpha}(x_{1},\ldots,x_{n})$$

where $(a)_{\kappa} = \prod_{(i,j)\in\kappa} (a - (i-1)/\alpha + j - 1)$ is the generalized Pocchammer symbol and C_{κ}^{α} is the "C" normalization of the Jack function, has numerous theoretical applications in multivariate statistical analysis, random matrix theory, etc.

Yet, it has been notoriously difficult to compute or approximate numerically, which has limited its practical importance.

In this talk we present a new algorithm which exploits the combinatorial properties of the Jack function and computes the truncation of ${}_{p}F_{q}^{\alpha}$ for $|\kappa| \leq m$ in time that is only linear in n and subexponential in m.



Figure 1: The p.d.f. of the smallest eigenvalue of a β -Laguerre matrix—experimental data and theoretical prediction

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