Plamen Koev
NCSU
232B Harrelson
Raleigh, NC 27695, U.S.A.

Time requested: 40
`pskoev@ncsu.edu`
`www4.ncsu.edu/ pskoev`
tel: +1-510-290-0752, fax: +1-484-450-5784
6 years since degree

Computing Eigenvalues of Random Matrices

Plamen Koev

While the subject of numerically computing the eigenvalues of matrices is quite mature, the problem of computing eigenvalue *distributions* of *random* matrices is still in its infancy. The problem is to compute the distributions of the eigenvalues given the distributions of the matrix entries for certain random matrix classes, e.g., Wishart, Jacobi, etc.

Somewhat surprisingly, explicit formulas for these eigenvalue distributions have been available since the 1960s, but only in terms of the *hypergeometric function of a matrix argument* – a notoriously slowly converging series of multivariate orthogonal polynomials called Jack functions. The efficient evaluation of such distributions had eluded the researchers for over 40 years even though the matrices involved are sometimes as small as 3×3.

The proposed talk will be accessible to anyone with numerical linear algebra knowledge as I will present the (fairly straightforward) derivation of the explicit formulas for the eigenvalue distributions of random matrices. In the main part of the talk, I will present the connections to combinatorics, representation theory and fast Fourier transforms, which we used to recently develop the first efficient algorithms for these distributions.

Our new algorithms had immediate effect to applications in multivariate statistics as techniques that had long been developed in theory could finally be used in practice: Multivariate statistics is concerned with the extraction of meaningful information from enormous datasets, most often organized as a matrix X of n observations on m objects. Various questions of correlation, interdependency, and classification of the data are critical in applications ranging from genomics to biostatistics and wireless communications to image and target recognition.

As a mathematical model for the above setup one considers a matrix of normal entries with arbirtary covariances $A \sim N_m(0, \Sigma)$. The matrix A^TA is called Wishart.

The statistics used to test various hypothesis on the original datasets are usually the extreme eigenvalues (or functions thereof) of the matrix X^TX. They are tested against the theoretical predictions for the Wishart matrix A^TA.

Thus the importance of being able to compute the theoretically predicted eigenvalue distributions.

The applications that have been enabled by our new algorithms range from genomics to target classification which I will also briefly describe.

I will also outline a number of prominent computational problems that remain open.