Total Positivity, Bidiagonal Decompositions, and Variation-Diminishing Properties

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Abstract

In this talk I will concentrate on the properties on the totally positive matrices, i.e., the matrices with all minors positive. Famous examples include the Pascal, Hilbert, and Vandermonde matrices (as long as the nodes of the latter are positive and increasing). These matrices possess very elegant structure going well beyond the positivity of the minors. For example:

- despite the general nonsymmetry, all eigenvalues are real and positive;
- the multiplication by a totally positive matrix does not increase the number of sign changes in a vector;
- the eigenvector corresponding to the jth largest eigenvalue has exactly $j-1$ changes of sign.

How can one establish these properties systematically?

Answer: By considering the bidiagonal decompositions of the totally positive matrices.

This (old) idea has become particularly prominent recently and has allowed for an easy and straightforward insight into many theoretical properties of these matrices.

I will survey the key ideas behind these bidiagonal decompositions and will demonstrate how easy it is to establish classical results such as the positivity of the eigenvalues or the variation-diminishing properties. I will conclude with some new results, including a full characterization, in the language of bidiagonal decompositions, of the eigenvector matrices of totally positive matrices.