The Efficient Computation of Multivariate Statistics though the Hypergeometric Function of a Matrix Argument.

Plamen Koev, Alan Edelman

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139

Various statistics of random matrices are expressible in terms of the *hypergeometric function of a matrix argument* X:

$$_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; X) \equiv \sum_{k=0}^{\infty} \sum_{\kappa \vdash k} \frac{(a_1)_\kappa \cdots (a_p)_\kappa}{k!(b_1)_\kappa \cdots (b_q)_\kappa} \cdot C_\kappa(X),$$

where $C_\kappa(X)$ is the zonal polynomial and $(a)_\kappa$ is the Pochhammer symbol.

For example, the comulative density function of the largest eigenvalue of an $n \times n$ Wishart matrix with l degrees of freedom and covariance matrix Σ, $A = W_n(l, \Sigma)$ is:

$$P(\lambda_A < x) = \frac{\Gamma_n\left(\frac{n+1}{2}\right)}{\Gamma_n\left(\frac{l+n+1}{2}\right)} \cdot \det\left(\tfrac{1}{2}x\Sigma^{-1}\right)^{l/2} \cdot {}_1F_1\left(\tfrac{l}{2}; \tfrac{n+l+1}{2}; -\tfrac{1}{2}x\Sigma^{-1}\right), \tag{1}$$

where Γ_n is the multivariate Gamma function. The distributions of the trace, the smallest eigenvalue, etc., of Wishart, Laguerre, etc., random matrices are also expressible in terms of $_pF_q$.

In this talk we present a new, efficient algorithm for evaluating $_pF_q$, which makes it possible to evaluate expressions like (1) very quickly and efficiently.

We briefly discuss the combinatorial aspects of the design of our algorithm as well as various examples and applications.