Accurate Eigenvalue and QR Decompositions of Totally Positive Matrices

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A matrix all of whose minors are positive is called *totally positive* (TP).

For computational purposes, the TP matrices are best represented not by their entries, but as products of positive bidiagonals. This representation allows highly accurate computations: eigenvalues, SVD, inverses, etc., can be computed to high relative accuracy. Using this representation one can perform accurate computations with derivative TP matrices, such as products, Schur complements, and submatrices of TP matrices.

In this talk we present our latest results: An accurate algorithm that computes eigenvectors of a TP matrix with the correct *combinatorial properties*: the jth computed eigenvector will have exactly $j-1$ sign changes.

We also demonstrate that the matrix Q of the QR decomposition of a TP matrix has the same variation of sign properties as the eigenvector matrix, and present an algorithm that would guarantee these in floating point arithmetic as well.