

18.336 Practice Midterm, March, 2005 (Open everything)

1. Show that if one takes $\lambda = k^{1/2}$, then the forward-time central space scheme is stable and consistent.
2. Show that the phase velocity for the Crank–Nicolson scheme is given by

$$\tan\left(\frac{1}{2}k\alpha(h\xi)\xi\right) = \frac{1}{2}a\lambda \sin h\xi$$

and satisfies

$$\alpha(h\xi) = a\left(1 - \frac{(h\xi)^2}{6}\left(1 + \frac{1}{2}a^2\lambda^2\right) + O(h\xi)^4\right).$$

3. Show that the group velocity for the Crank–Nicolson scheme is given by

$$\gamma(\theta) = a\frac{\cos\theta}{1 + \frac{1}{4}a^2\lambda^2 \sin^2\theta}.$$