

18.336, Homework # 6, Due 5/11/2005

1. Determine the eigenvalues of the Jacobi iteration matrix when applied to the “diagonal” five-point scheme given by

$$\frac{1}{h^2}(v_{m-1,l-1} + v_{m-1,l+1} + v_{m+1,l-1} + v_{m+1,l+1} - 4v_{ml}) = f_{ml}$$

on a uniform grid with $\Delta x = \Delta y = h$.

Hint: The eigenvectors are the same as the ones for the usual Jacobi method.

2. Using SOR(w) solve

$$u_{xx} + u_{yy} = -2 \cos x \sin y$$

on the unit square. The boundary conditions and the exact solutions are given by $u = \cos x \sin y$. Plot the solution.

Demonstrate that convergence takes time that is linear in $N = 1/h$. Namely show that it takes $O(N)$ steps for the difference between two consecutive iterates (measured in L^2 norm as $\|e\| = (\sum_{m,l} |e_{ml}|^2 h^2)^{1/2}$) to become (say) 10^{-6} .