# 18.336, Homework # 4, Due 4/12/2005

1. Find the amplification factor for the Crank-Nicolson scheme for $u_t = bu_{xx}$ and decide if the scheme is stable and/or dissipative.

2. Solve $u_t = bu_{xx}$ on $-1 \le x \le 1$ with initial data

$$u_0(x) = \begin{cases} 1 & \text{if } |x| < \frac{1}{2} \\ \frac{1}{2} & \text{if } |x| = \frac{1}{2} \\ 0 & \text{if } |x| > \frac{1}{2}. \end{cases}$$

   Solve up to $t = \frac{1}{2}$. The boundary data and the exact solutions are given by

$$u(t,x) = \frac{1}{2} + 2\sum_{l=0}^{\infty} (-1)^l \frac{\cos \pi(2l+1)x}{\pi(2l+1)} e^{-\pi^2(2l+1)^2 t}.$$

   Use the Crank–Nicolson scheme

$$\frac{v_m^{n+1} - v_m^n}{k} = \frac{b}{2}\left( \frac{v_{m+1}^{n+1} - 2v_m^{n+1} + v_{m-1}^{n+1}}{h^2} + \frac{v_{m+1}^n - 2v_m^n + v_{m-1}^n}{h^2} \right)$$

   with $h = \frac{1}{10}, \frac{1}{20}, \frac{1}{40}$. Compare the accuracy and efficiency when $\lambda = 1$ and also when $\mu = 10$.

   Demonstrate by the computations that when $\lambda$ is constant, the error in the solution does not decrease when measured in the supremum norm, but it does decrease in the $L^2$ norm.