18.336, Homework # 3, Due 3/9/2005

1. Using MATLAB solve $u_t + au_x = 0$ on $-1 \le x \le 9$ for $0 \le t \le 8$ with $a = 1,\ \xi_0 = 5\pi$ and initial condition

$$u(0,x) = \begin{cases} \cos \xi_0 x \cdot \cos^2 \frac{x\pi}{2}, & |x| \le 1; \\ 0, & \text{otherwise} \end{cases}$$

using the Lax–Wendroff scheme

$$v_m^{n+1} = v_m^n - \frac{a\lambda}{2}(v_{m+1}^n - v_{m-1}^n) + \frac{a^2\lambda^2}{2}(v_{m+1}^n - 2v_m^n + v_{m-1}^n),$$

with $\lambda = 0.95,\ h = 0.05$ and zero boundary conditions.

Derive the following formula for the phase velocity:

$$\alpha(\theta) = a\left(1 - \frac{1}{6}\theta^2(1 - a^2\lambda^2) + O(\theta^4)\right).$$

Demonstrate that the wave packet moves with the group velocity and the ξ_0-mode travels with the phase velocity. In this exercise you will, of course, see the effects of dissipation on the solution.