

18.336, Homework # 2, Due 3/2/2005

1. Show that the following scheme is consistent with $u_t + au_x = 0$.

$$\frac{v_m^{n+1} - v_m^n}{k} + \frac{a}{2} \left(\frac{v_{m+1}^{n+1} - v_m^{n+1}}{h} + \frac{v_m^n - v_{m-1}^n}{h} \right) = 0.$$

2. Show that the scheme

$$\frac{v_m^{n+1} - v_m^n}{k} + a \frac{v_{m+2}^n - 3v_{m+1}^n + 3v_m^n - v_{m-1}^n}{h^3} = 0$$

is consistent with $u_t + au_{xxx} = 0$ and if $\nu = kh^{-3}$ is constant, then it is stable when $0 \leq a\nu \leq \frac{1}{4}$.

3. Use the unstable forward-time central-space scheme with the following two sets of initial data on the interval $[-1, 3]$ for $0 \leq t \leq 1$:

- $u_0(x) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0, & \text{otherwise.} \end{cases}$
- $u_0(x) = \sin x$

Use $a = 1$, $h = 0.1$, and $\lambda = 0.8$. Demonstrate that the instability is evident sooner with the less smooth initial data than it is for the smooth data. Show that the growth of the instability for each case is about $g(\pi/2)$.