

Lecture # 2

- Outline:
- 1) Review from last time
 - 2) A word on Mathematical rigor
 - 3) Surfaces
 - 4) Polygonal Complexes

Section 1 : Review

Remark: 1) Topology = the study of shape

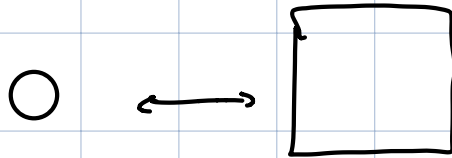
2) Two objects have the same shape if we can continuously deform / rearrange, in an invertible manner, the points of one object to obtain the other object.

↳ Continuous : takes points infinitesimally close together to points infinitesimally close together

↳ Invertible : Continuous undoing of rearrangement such that every point goes back to where it started.

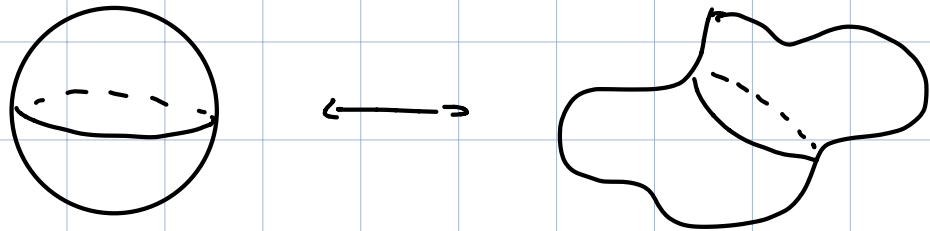
3) If two spaces/ objects have the same shape, then we say they are homeomorphic

↳ Small circle vs Big square



- size doesn't matter
- angles don't matter

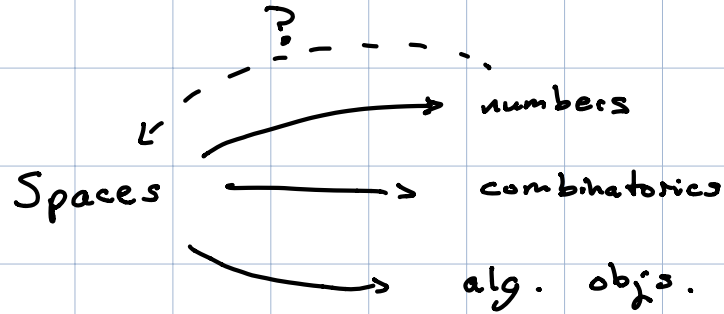
↳ Deflated Beach Ball vs Round Beach Ball



↳ Coffee Mug vs Donut

Goal : 1) create invariants of spaces that detect differences in shape

2) Use invariants to prove non-trivial results



Section: Mathematical Rigor

Remark: Need MATHEMATICAL means of studying shapes as oppose to a heuristical/visual means.

- ↳ There are spaces we can't visualize, but that we can nevertheless study.

Claim: The square root of 2 is irrational.

↳ $\sqrt{2} \neq p/q$ for some integers p and q

Remark: The proof goes via a technique called proof by contradiction

- Remark:
- A statement is either true or false.
 - To prove true, either prove that it is true or not false.
 - To prove not false, we assume that the false statement holds and then under this assumption try to show that something we know (a priori) to be true to be false, under this assumption.
 - We say that we arrive at a contradiction.
 - Upshot is that our original assumption of falsehood was incorrect. So the statement must have been true.

⑤

Proof:

- Spse by way of contradiction that $\sqrt{2} = p/q$.
- We may assume $p = p_1 \dots p_n$, $q = q_1 \dots q_e$ w/ no common prime factors (ie p/q is in lowest-terms)

- $\Rightarrow 2q^2 = p^2$

- $\Rightarrow p^2$ is even

- $\Rightarrow p$ is even

square of
odd is odd

- $\Rightarrow p^2$ is divisible by 4, so $p^2 = 4 \cdot r$

- $\Rightarrow q^2 = 2r$

- $\Rightarrow q^2$ is even

- $\Rightarrow q$ is even

- So both p and q have a common factor of 2

\Rightarrow contradiction

□

Remark: • Why does higher math always appear so alien?

↳ Fact: Locally near 0, there exists holomorphic coordinates w_1, \dots, w_n st

$$\Omega_s = \sum_{i,j} \frac{i}{2} (\delta_{i\bar{j}} + f_{i\bar{j}}^s) \cdot dw_i + d\bar{w}_j$$

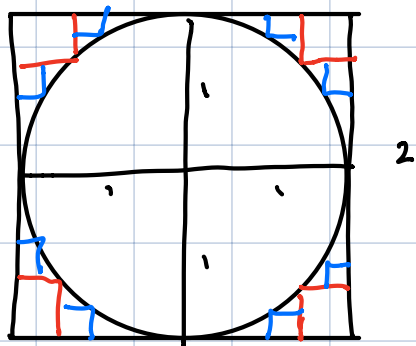
where each $f_{i\bar{j}}^s$ is a function that vanishes to first order at the origin.



↳ the 1-jets of $f_{i\bar{j}}^s$ vanish at the origin.

- Jargon, notation, technical definitions, etc. allow mathematicians to concisely express and rigorously prove ideas.
- For this class, Ideas/pictures \succ technical details.

Claim: $\pi = 4$

Proof:



- Circum = 2π
 - perim $\square = 8$.
 - perim  = 8.
 - perim.  = 8
- ↓ ∞ # of times
- $\Rightarrow 2\pi = 8$
- $\Rightarrow \pi = 4$ □

Section: Surfaces

Definition: A surface is space that locally looks like \mathbb{R}^2
↳ ie, zoom in close it just looks like a piece of paper.

↳ homeomorphic to unit disk in \mathbb{R}^2 .

Examples: ① Beach ball = sphere = S^2 .

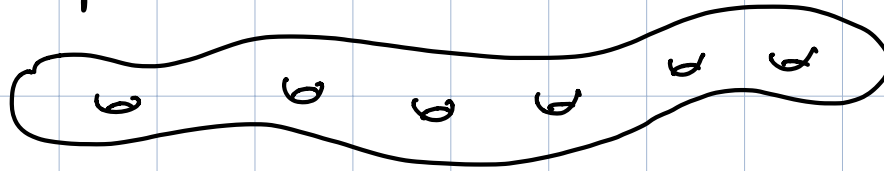


② Intertube = torus = T^2

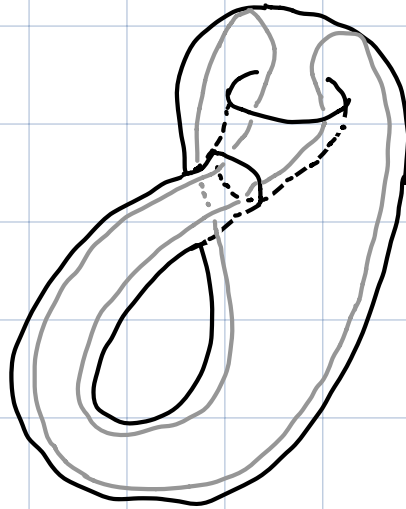


③

③ Complicated intertube



④ Klein Bottle



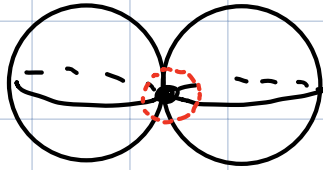
↪ actually need 4-dimensions to realize as surface, ie,
to resolve this "crossing"

⑨

non-example

⑤

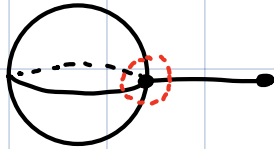
Kissing spheres



non-example

⑥

Ball on a stick



Remark:

We want to view surfaces as combinatorial objects.

↳ We think about our globe as a map, which is combinatorial.

Our first attempt to study shape will be to turn

⑩

topology into combinatorics.

Section : Polygonal Complexes

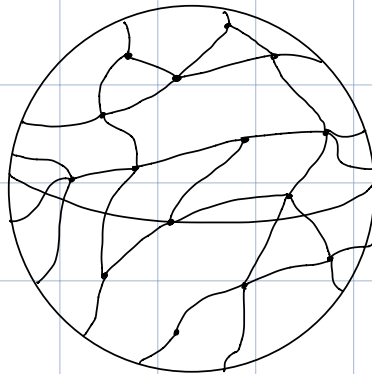
Example :

Divide the globe up according to regions.

That is, envision globe as polygons patched together

- ① Faces of polygons \rightarrow regions
- ② Edges of polygons \rightarrow borders
- ③ Vertices of polygons \rightarrow where regions meet

Picture :



Remark:

This works equally as well w/ any surface

Remark:

We will consider a broader class of spaces called polygonal complexes.

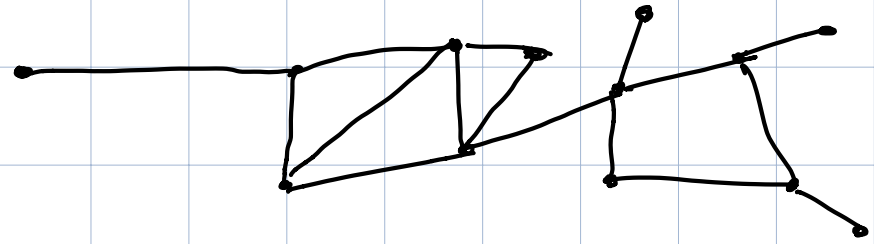
↳ surfaces broken up into polygons will give a specific class of examples.

Definition:

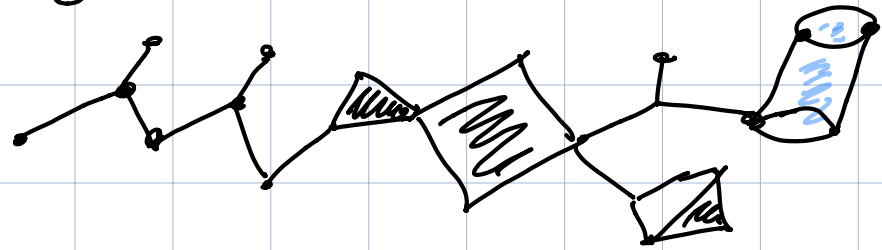
A polygonal complex is a space obtained by gluing together polygons, edges, and vertices, where by glue we mean that we identify edges w/ edges and vertices w/ vertices (could glue polygon to self)

Example:

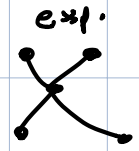
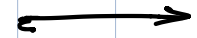
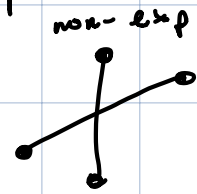
① Graph



② Something wild



③ Non-example

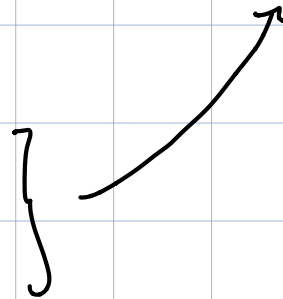


3) Mobius band



} direction that we glue
edges together matter

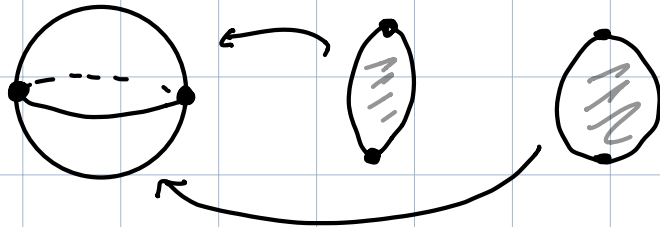
4) Cylinder



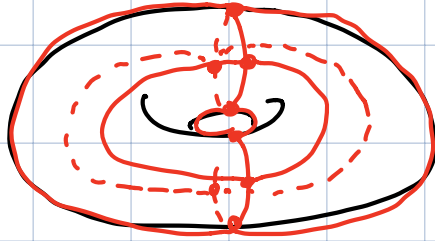
5) Sphere



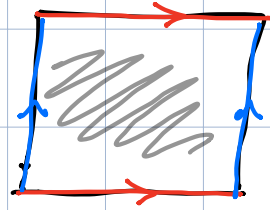
or



⑥ Torus



or

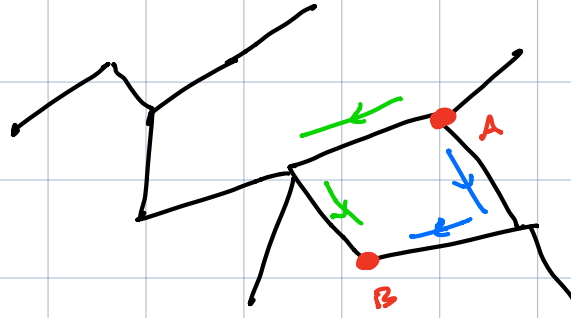


- Definition:
- A graph is a polygonal complex composed of edges.
 - A graph is a tree if every pair of vertices is connected via a unique sequence of edges.

↳ no loops in the graph.

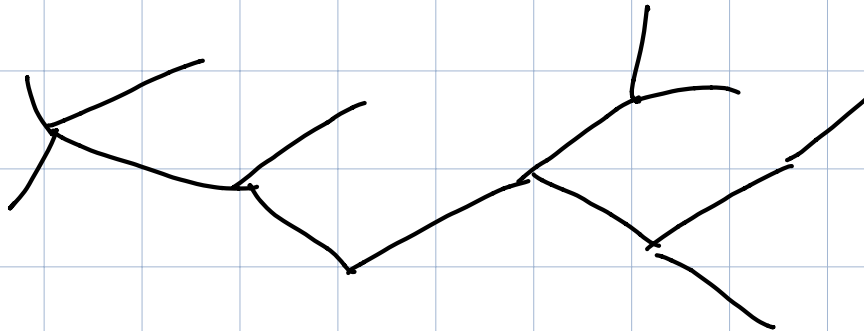
Example:

1)



not tree

2)



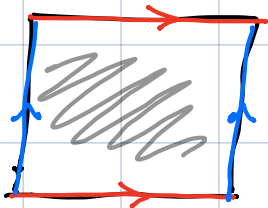
tree

Definition: A planer diagram is a polygonal complex obtained by gluing together all pairs of edges of a single $2n$ -polygon.

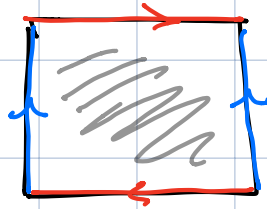
Example: 1) Sphere



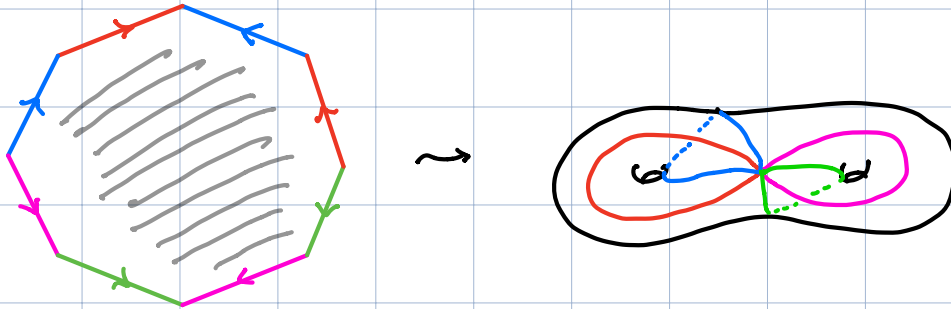
2) Torus



3) Klein bottle



4) Genus 2 surface



- Remark: 1) We can always break surfaces up into polygonal cpxes
↳ A surface is homeomorphic to this associated polygonal complex
- 2) There are an infinite # of ways we could break it up
- 3) There are strictly infinitely many more polygonal cpxes than surfaces.

- Next time:
- 1) We will prove that the Euler characteristics of any two polygonal cpxes associated to a surface agree
 - 2) Study Euler characteristics of graphs
 - 3) Planarity of graphs
 - 4) Colorings of Maps Theorem

