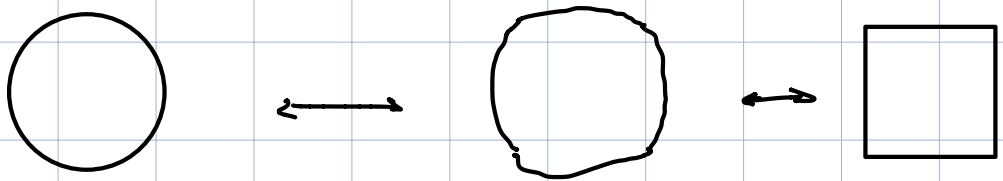
Lecture #1: An Introduction

Remark: Topus = shape
ology = study of
} Topology = study of shape

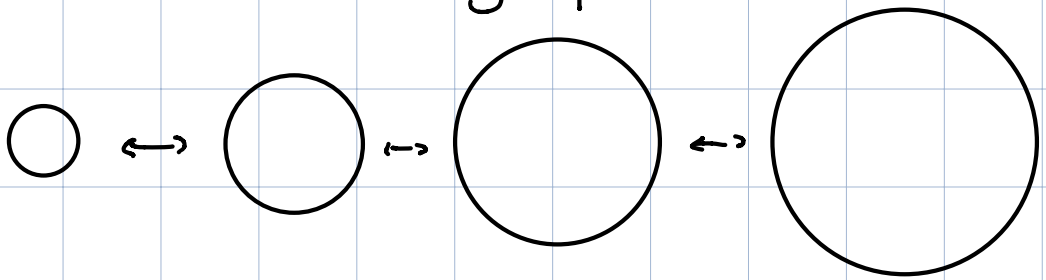
Question: What does it mean for things to have the same shape?

Answer: Two objects have the same shape if we can continuously deform or rearrange the points in one obj to obtain the other object.

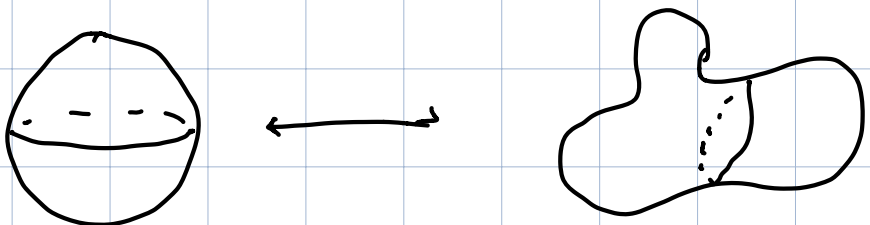
Examples: ① Circle versus a square (angles)



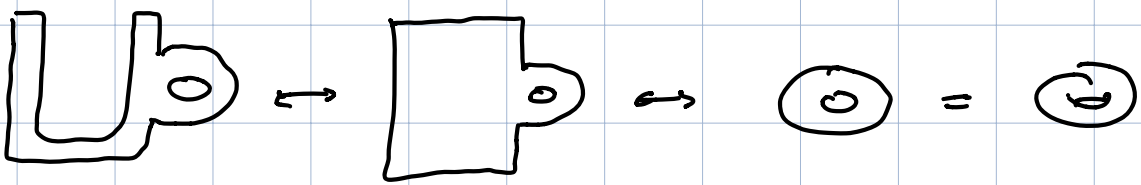
② Small square versus Big square (size)



③ Deflated Beach Ball

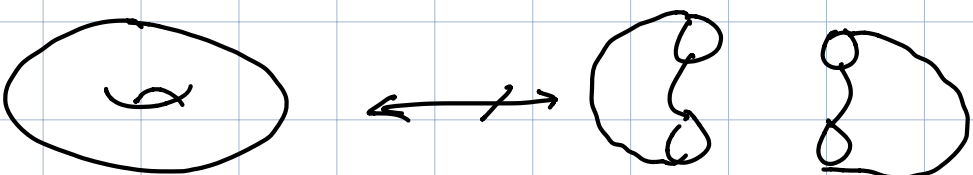


④ Coffee Mug versus a Donut



non-ex. ⑤ Circle versus an interval (gluing)

non-ex ⑥ Sharing a Donut (ripping / cutting)



Remark: Two objects have the same shape if these deformings / rearrangings that relate them are

1) Invertible

↳ There is a way of undoing the rearrangement such that every point goes back to where it started.

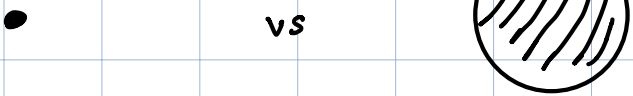
2) Continuous

↳ The rearrangement takes points that are infintesimally close together to points that are infintesimally close together

Question: Do the following spaces/objects have the same shape?

①  vs

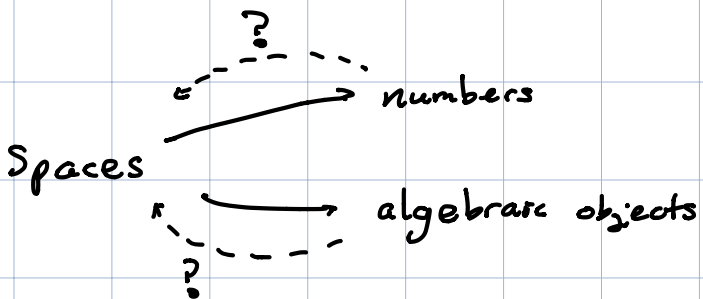
②  vs

③  vs

Definition: If two objects have the same shape, then we say that they are homeomorphic

Remark: Topologists try to create invariants of spaces that detect differences in shape.

↳



Example: The # of connected pieces/components

↳ If different # of pieces, then not homeomorphic.

Remark: We will learn about other invariants:

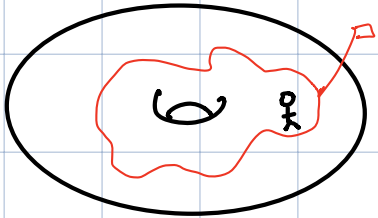
1. Euler characteristic
2. Degree of a map
3. Fundamental group
4. homology

Remark: Need MATHEMATICAL means of studying shapes as oppose to a heuristical/visual means.

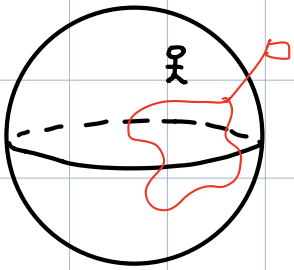
↳ There are spaces we can't visualize, but that we can nevertheless study.

Question: (A thought experiment) Aliens have abducted you and your friends and transported you to an alternate universe for experimentation. However, the man w/ a cat who runs the universe prohibits experimentation on "logical" beings. Thus the aliens set up a task for you to complete. They drop you on a planet and ask you to deduce if the planet has the shape of a ball or a donut (the physical laws of the universe allow for this possibility). You may request and use any tools you want. However, the aliens have cloaked the planet so outside observation from outerspace is useless.

Picture:



vs



↳ Are you an ant on a beach ball or an intertube

Question: (Prequestion) Do we expect a donut and a ball to have the same shape?

Topics Ahead:

1. Colorings of Maps Theorem
2. Classification of surfaces
3. Brouwer's Fixed Point Theorem ⇝ Nash?
4. Fundamental Theorem of algebra
5. Hairy Ball Theorem
6. Persistence and Topological Data Analysis
7. Knot Theory
8. Some other popular demand?