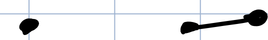
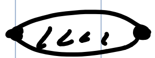
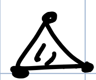
Lecture #2



Defn: A polygonal cpx is a gluing of vertices, edges, and n-polygons where glue means glue edges to edges along boundaries of polygons.

↳ n-poly = disk w/ n-sides

2-poly = 

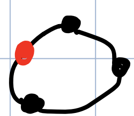
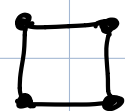
3-poly = 

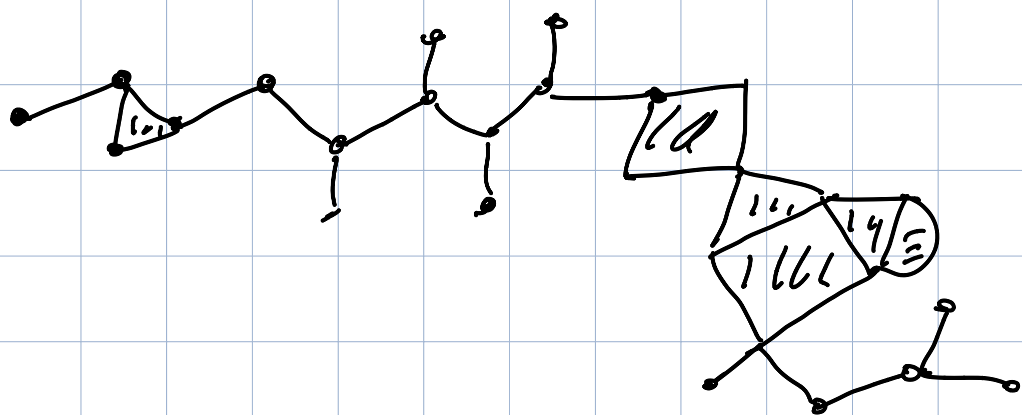
4-poly = 

5-poly = 

Ex:

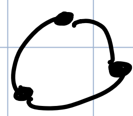
i) 

ii)  ≃ circle ≃  =: S^1

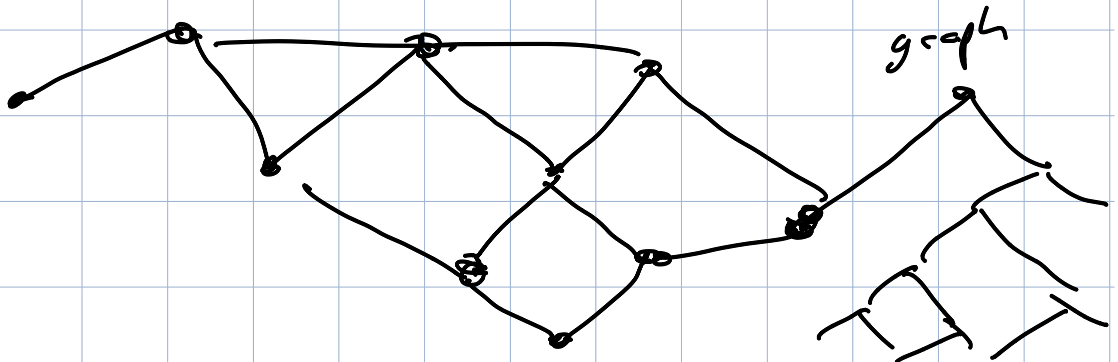
iii) 

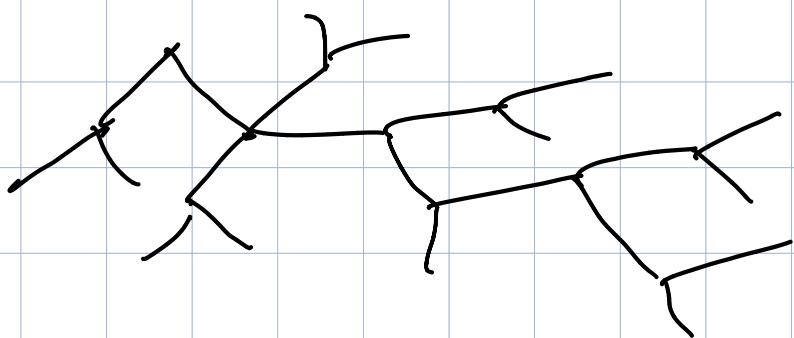
Defn: A graph is a polygonal cpx composed entirely of edges. A graph is a tree if each pair of vert may be joined via a unique path of edges

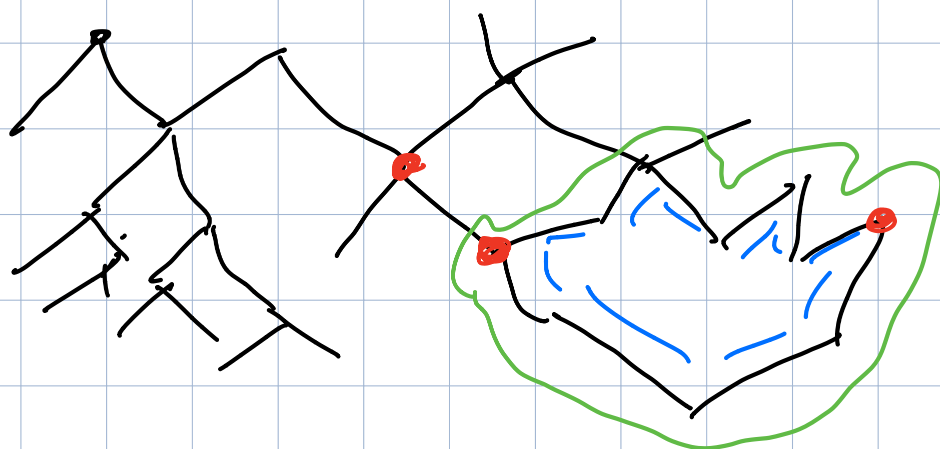
Ex:

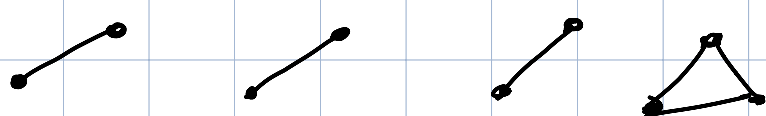
i) 

graph

ii) 

iii) 

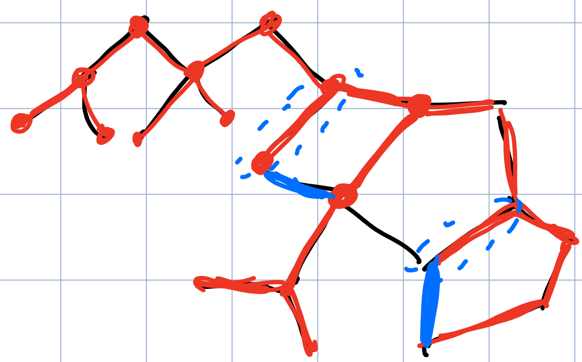
iv) 

v) 

Rmk: Tree is a graph w/ no loops.

Defn: A spanning tree of a conn. graph is a subcollection of edges that form a conn tree and contain all the vertices of the graph.

Ex: i)



ii)

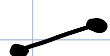
Rmk: There can exist mult diff. spanning associated to any graph.

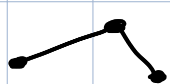
Prop: Every conn graph admits a spanning tree.

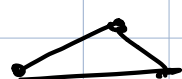
Proof: Build up the graph, Γ, in a seq of edge gluing. Denote this seq of cons. graphs by

$$\Gamma_0, \Gamma_1, \Gamma_2, \ldots, \Gamma_n = \Gamma$$

↪

$\Gamma_0 =$ 

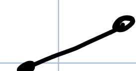
$\Gamma_1 =$ 

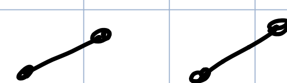
$\Gamma_2 =$ 

$\Gamma_3 =$ 

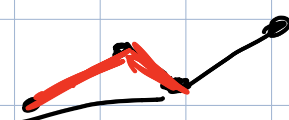
$\vdots$

Γ_n

$\Gamma_0 =$ 

$\Gamma_1 =$ 

$\Gamma_2 =$ 

$\Gamma_3 =$ 

↪ We can assume each Γ_i is connected as well.

For each Γ_i we construct a spanning tree T_i.

$T_0 = \bullet\!-\!\!\circ = \Gamma_0$

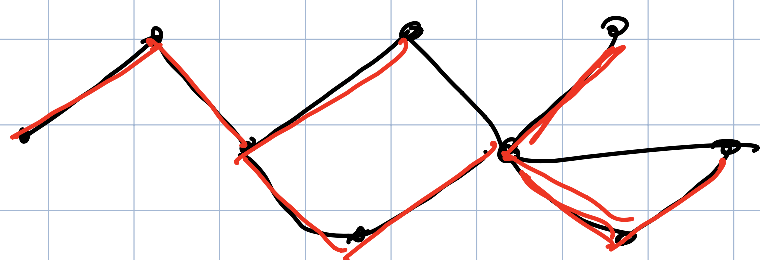
$\begin{cases} T_1 = T_0 & \text{if we don't add new vert to } \Gamma_1 \\ T_1 = \Gamma_1 & \text{if we add a new vert. to } \Gamma_1 \end{cases}$

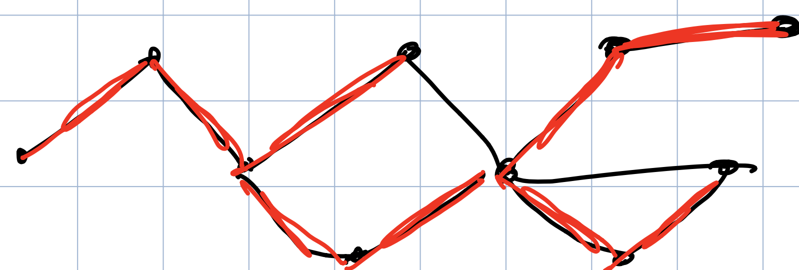
Spse that we know what the spanning tree is for Γ_i.

Say T_i.

Want to construct T_{i+1}

$\begin{cases} T_{i+1} = T_i & \text{if we don't add new vert to } \Gamma_{i+1} \\ T_{i+1} = T_i \text{ union the newly added edge} & \text{if we add a new vert. to } \Gamma_{i+1} \end{cases}$

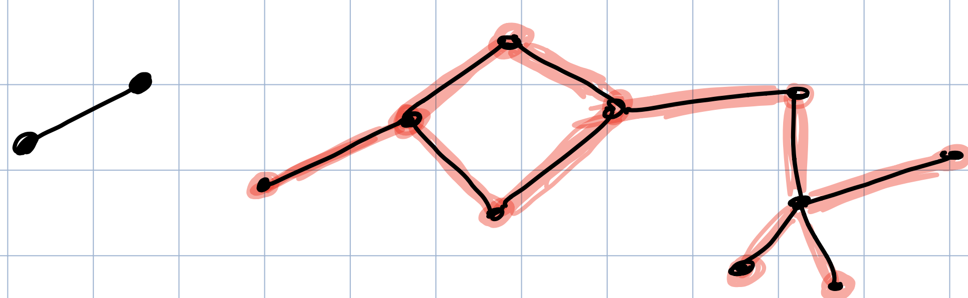




Q.E.D.

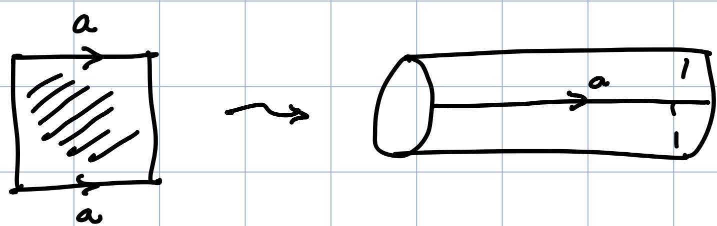
Rinse and repeat until $i = n$ and we are done! $\square$

Rmk:



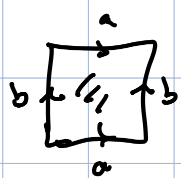
Defn: A planar dgm is a polygonal cpx obtained from gluing pairs of edges of a single $2n$-polygon.

Ex: i)



$\rightsquigarrow$ 2-sides

ii)

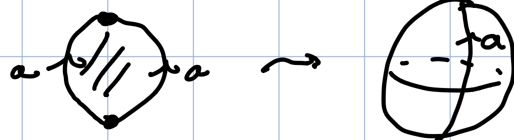


$\rightsquigarrow$ Torus =

$=: T^2$ = 2-torus

$:=$ defn to be equal

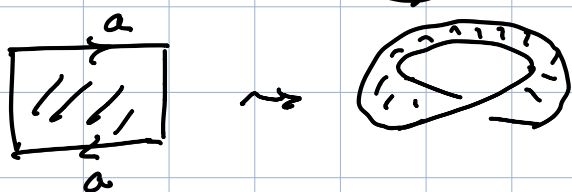
iii)



$=: S^2$ = sphere.

(S^1 = circle, T^1 = 1-torus)

iv)



$\rightsquigarrow$ 1-side

= Möbius band

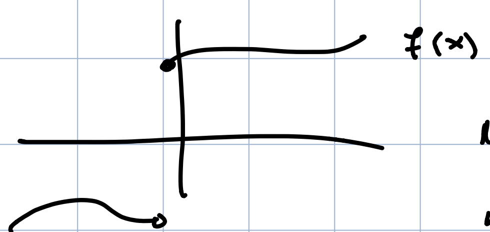
$f : \mathbb{R} \to \mathbb{R}$

$f(x) = y$

Defn: • A continuous map between topological spaces, $f : X \longrightarrow Y$, is an assignment of pts in X to points in Y st f takes pts infinitesimaly close together to pts inf close together.

$\hookrightarrow$ cont. map preserves the notion of closeness.

$\hookrightarrow$



$f(x)$, $f : \mathbb{R} \to \mathbb{R}$.

Not cont, ie, doesn't pres. notion of closeness.

$\hookrightarrow$ f :

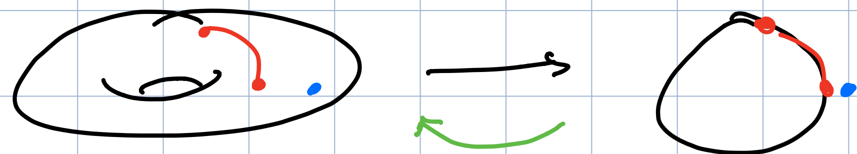
$\longrightarrow \mathbb{R}$

$f(\text{Northern hemi}) = 1$

$f(\text{southern hemi}) = 77$.

↳ $f:$  $\longrightarrow S^1$

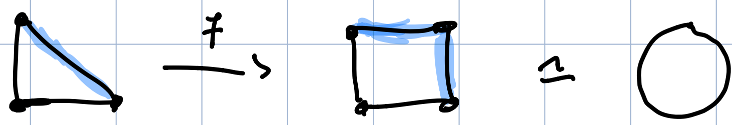
$f(x) =$ angle of x about the frosting direction.



(ii)

f^{-1} = Doesn't exist since it wouldn't be a fcn.

Yes it is cont.

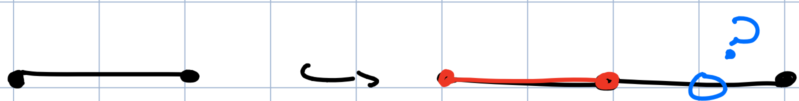
↳ 

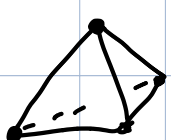
(i)

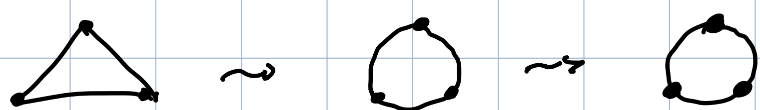
invert.

Defn: A homeomorphism is a continuous map $f: X \to Y$ that is a bijection.

↪ if it is invertible (go backwards, $f^{-1}: Y \to X$ st $f \circ f^{-1} =$ Identity, $f^{-1} \circ f =$ Id.)

Rmk: 

Ex:  ↗ sides fill, but hollow $\cong S^2$

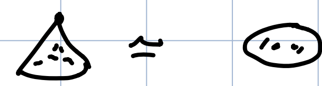


Defn: A surface is a top space that "locally" is homeo to an "open" disk in $\mathbb{R}^2$

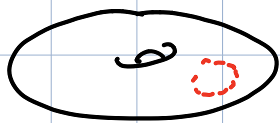
↪ about each point.

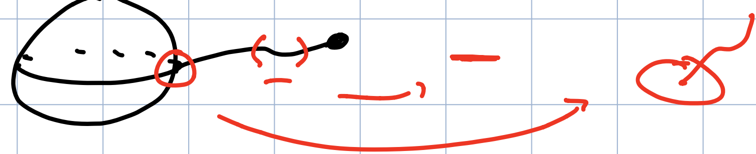
↳ $\{(x,y) \mid x^2+y^2<1\} =$ 

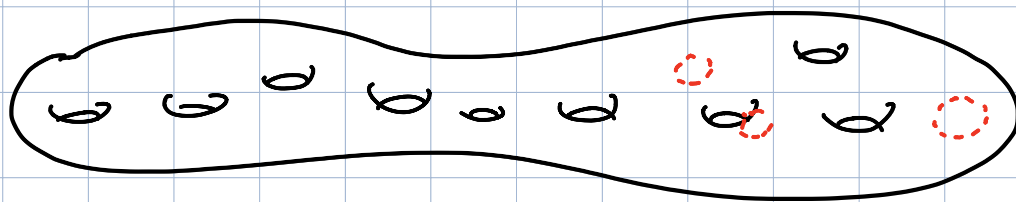
If you zoom in on the space it just looks like a piece of paper.

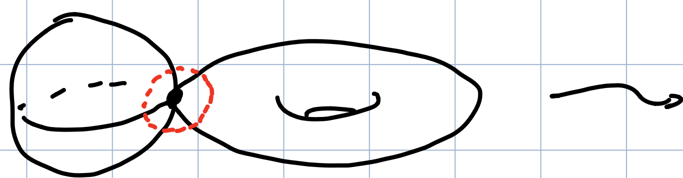
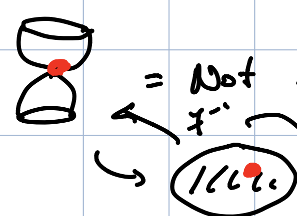
 → loc. piece of paper.

Ex: i)  $= S^2$

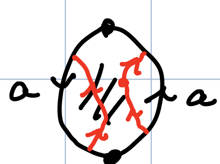
ii)  $= T^2$

iii)  Not surf.

iv) 

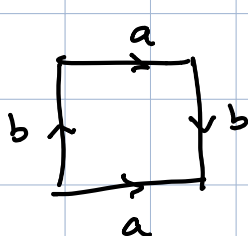
v)   = Not surf.

away from • = red pt cont.

Ex: i) 

= real projective space ($\mathbb{RP}^2$, P^2)

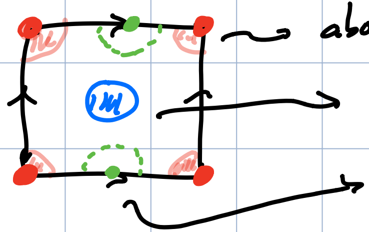
↳ can't be embedded in 3-dim space

ii) 

= Klein bottle = K

Prop: A planar dgm w/ all edges glued (not like cyclic) is a surface.

Proof:



about vertex we find some open disk!

about pts in poly int " " " " "

about " " edges