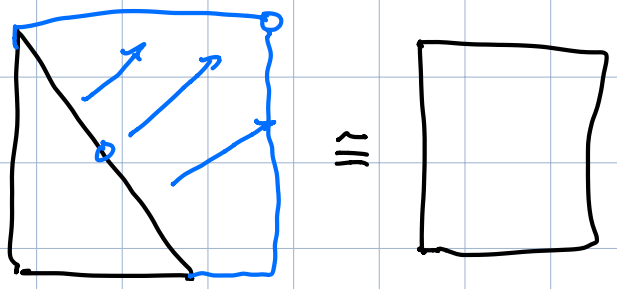
Lecture # 1

Rmk: Topology? = the study of shape

↪ Topos = latin for shape

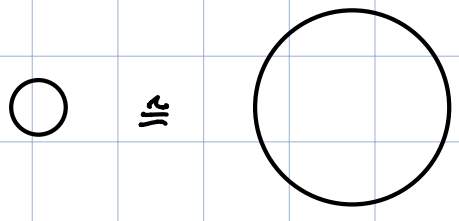
↪ logy = " " study of

Rmk: Two things have same shape if deform one to the other.

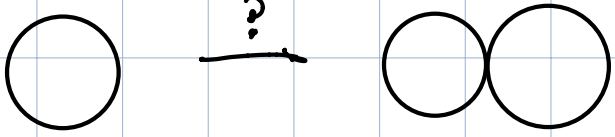


↪ Don't care about angles

↪ Don't care about size

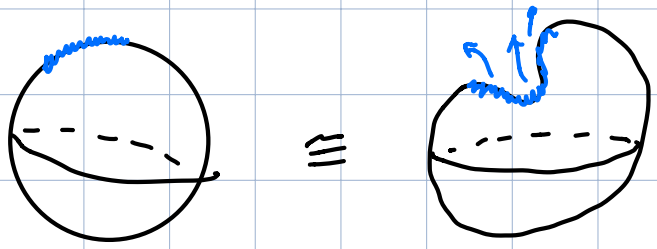


Ex: (Wan)

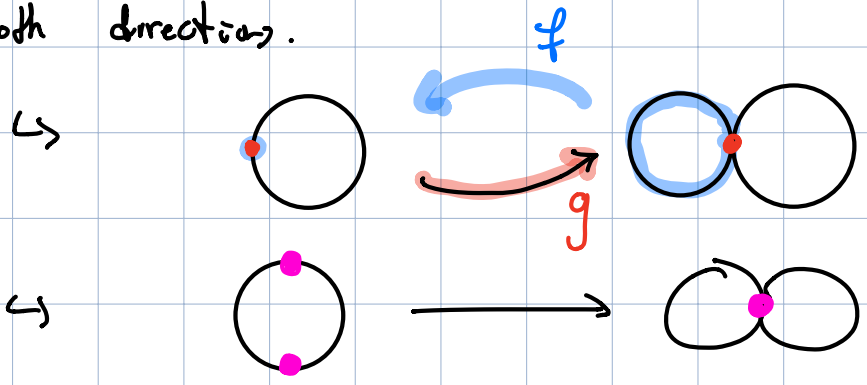


↪ We can't deform by ripping

Ex:

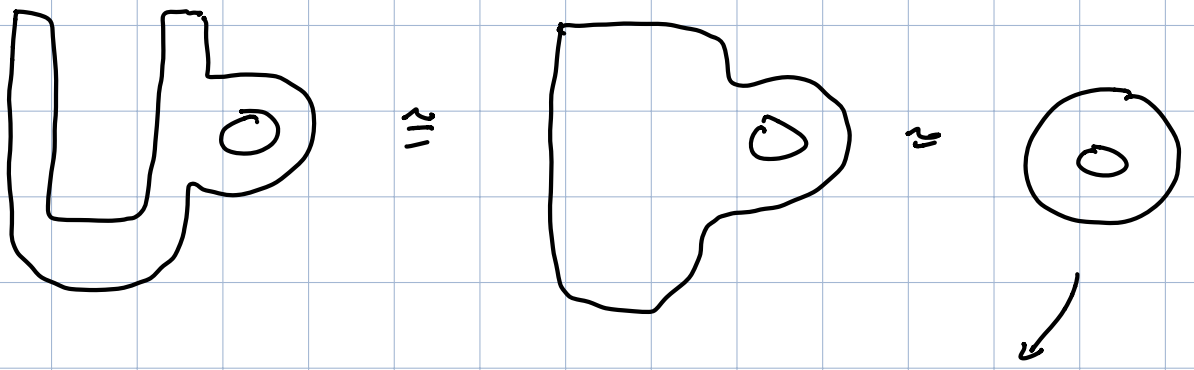


Rmk: These defs need to be fcns (assign pts to exactly one pt).
Same shape iff (if and only if) we have defs going both directions.



This does go the other way; however, we want $g \circ f = \text{identity}$, $f \circ g = id$.

Ex:



This is 2D - cross sec.
=> 3D is donut

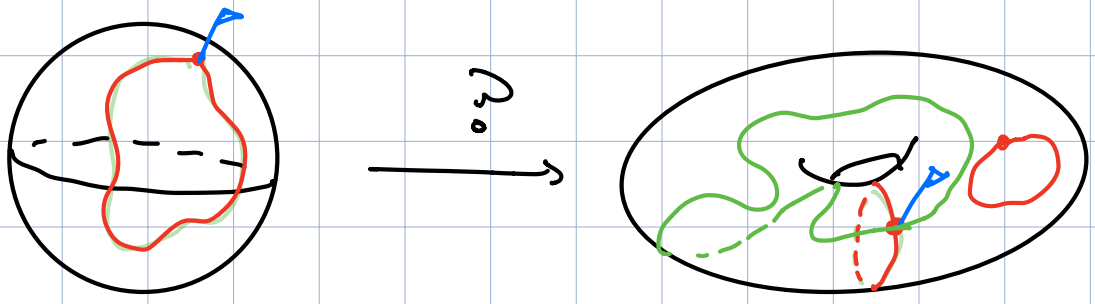
=

Goal: Assign invariants to objects / spaces that can distinguish shape

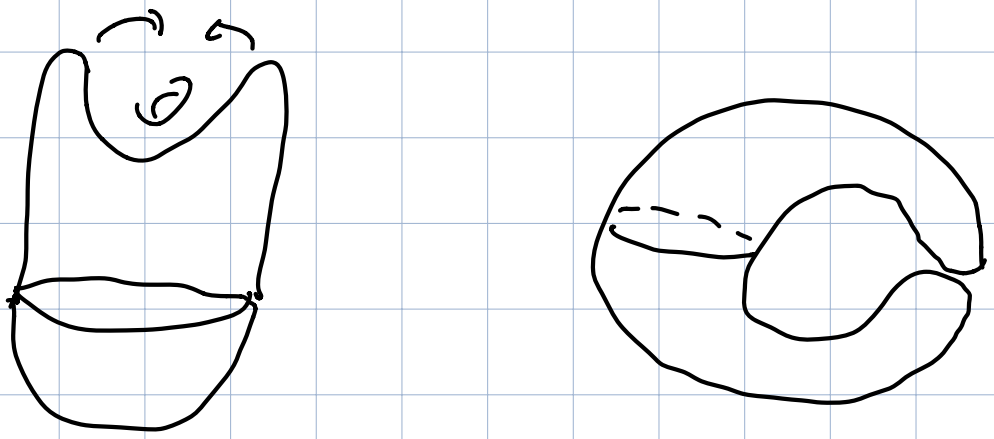
Spaces / Objects $\xrightarrow{\text{assign invariant}}$ Invariant

↳ inv. ind. of shape. (ie diff inv => diff shapes)

Ex:

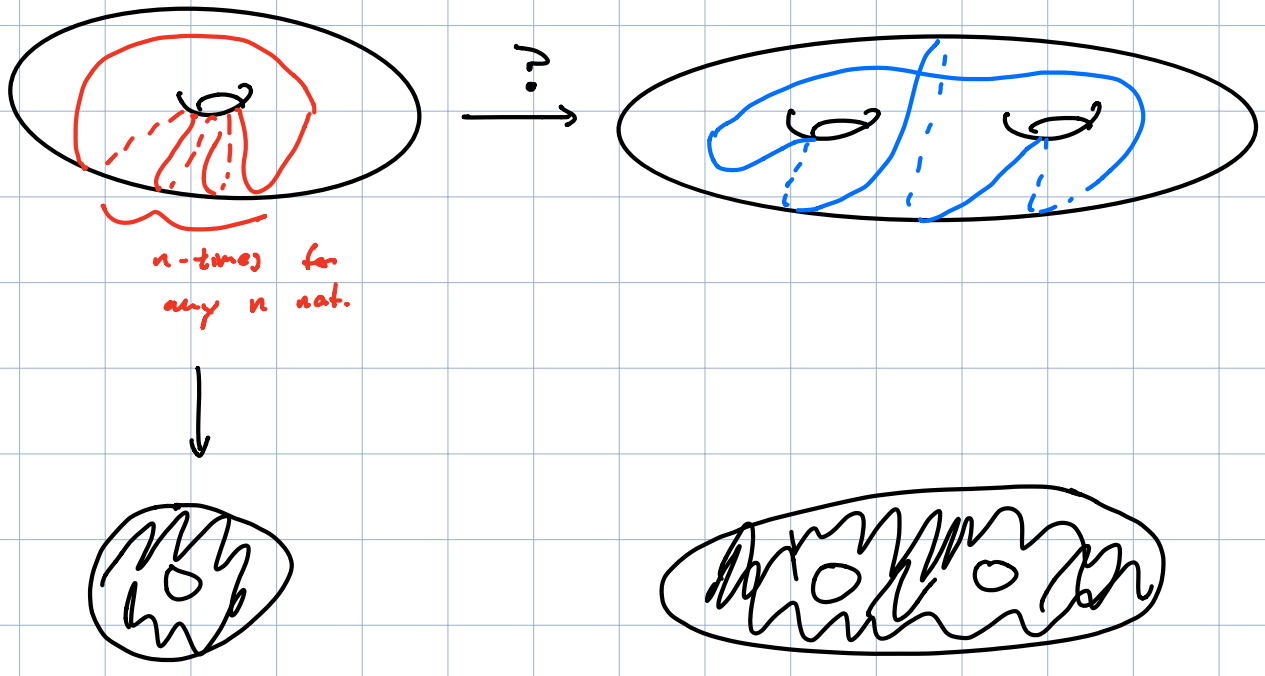


Do these have the same shape?



Rmk: (x,y) in $\mathbb{R}^2$ = coord. plane

Ex:



Defn: A topological space is a set of points w/ a notion of when two pts are "inf" close to each other.

Ex:

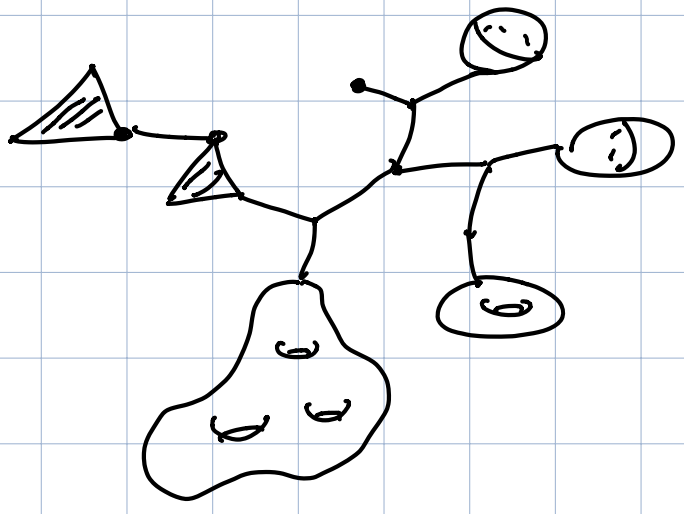
i)

ii)

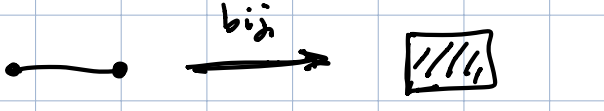
iii)

iv)

v)



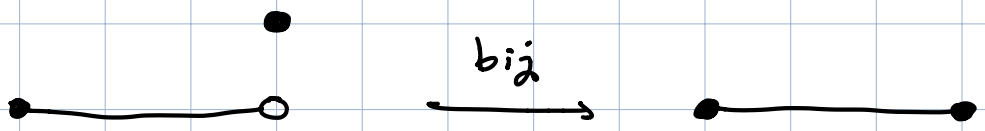
Ex:



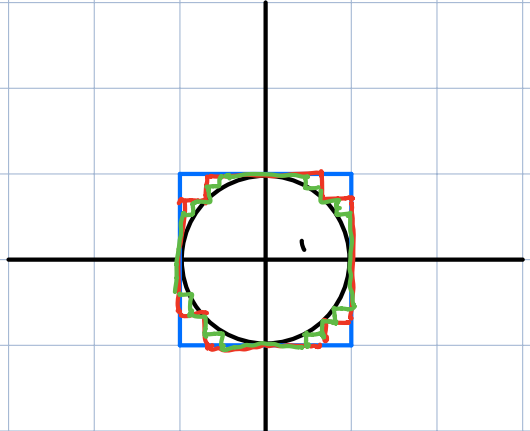
↳ not continuous

countable order $\mathbb{R}$

Ex:



Claim: $\pi = 4$



length of blue = 8

cir of circle = 2π

length of red = 8

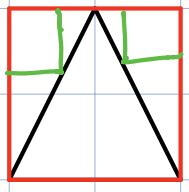
length of green = 8

Repeat process ⇝ curve that is inf. close to the circle but every step doesn't change length

⇒ length circle = 8

⇒ $8 = 2\pi$

⇒ $4 = \pi$ □



length red = 8

tri per = $2 + 2\sqrt{5}$

⇝ $8 = 2 + 2\sqrt{5}$

→ $3 = \sqrt{5}$.