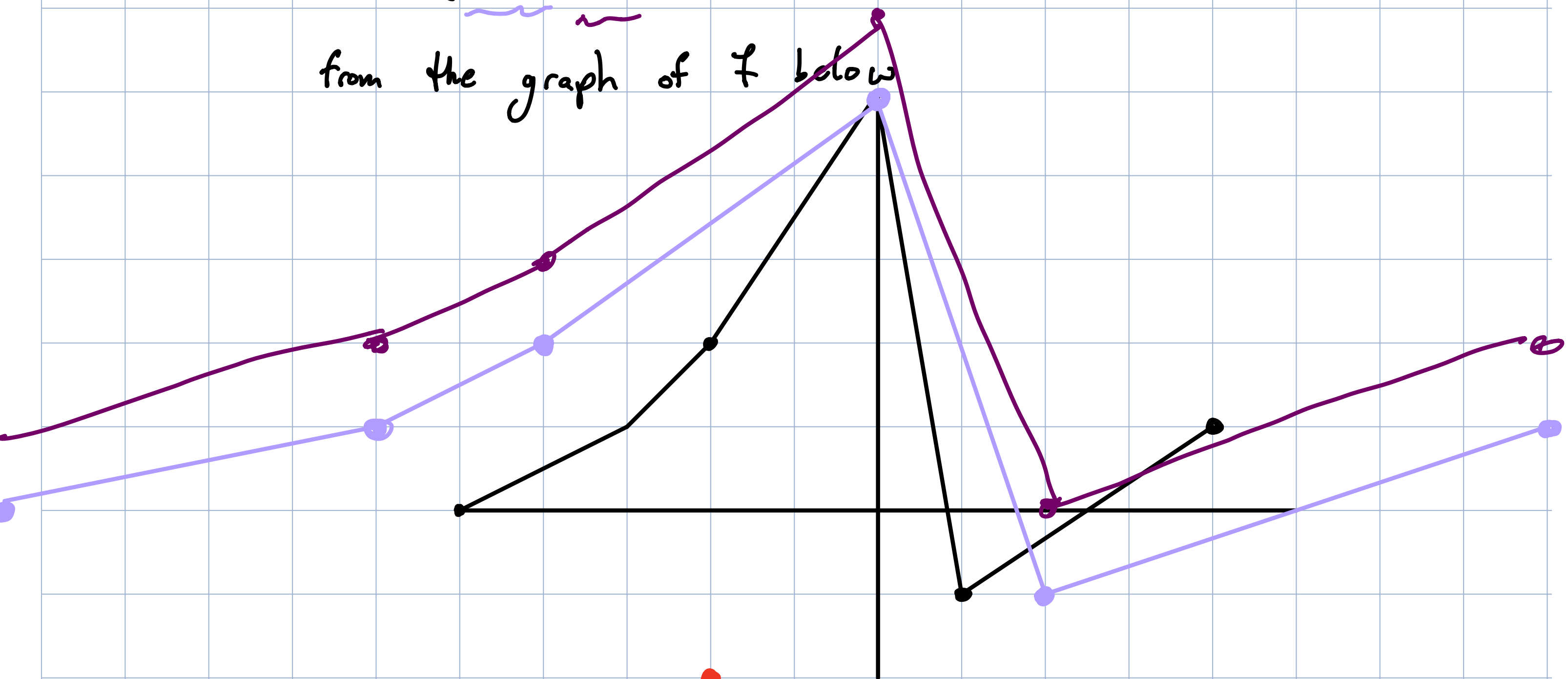
Lecture #9

Warm-up: 1) Sketch the graph of

a) $2f(x+1)-1$

b) $f(\frac{1}{2}x)+1$

from the graph of f below



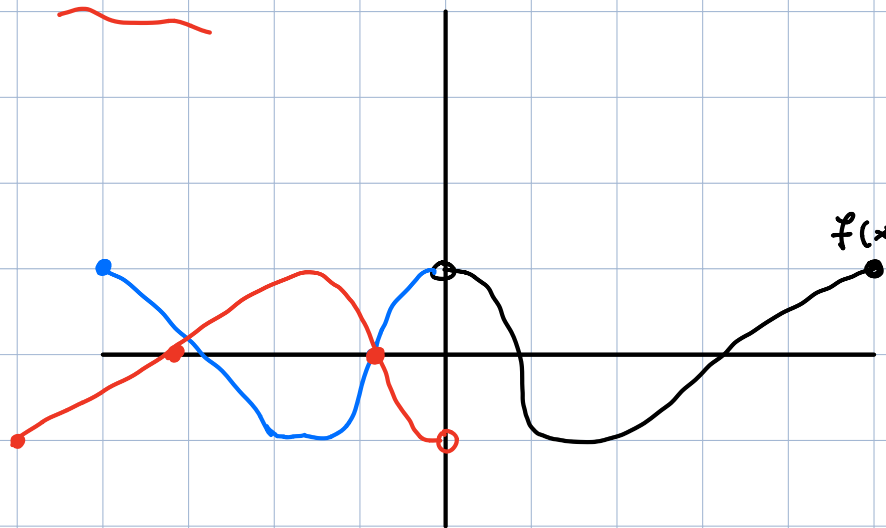
$f(x+1)-1$

$2(f(x+1)-1)$

2) Complete the graph of f below by assuming

a) f is even

b) f is odd

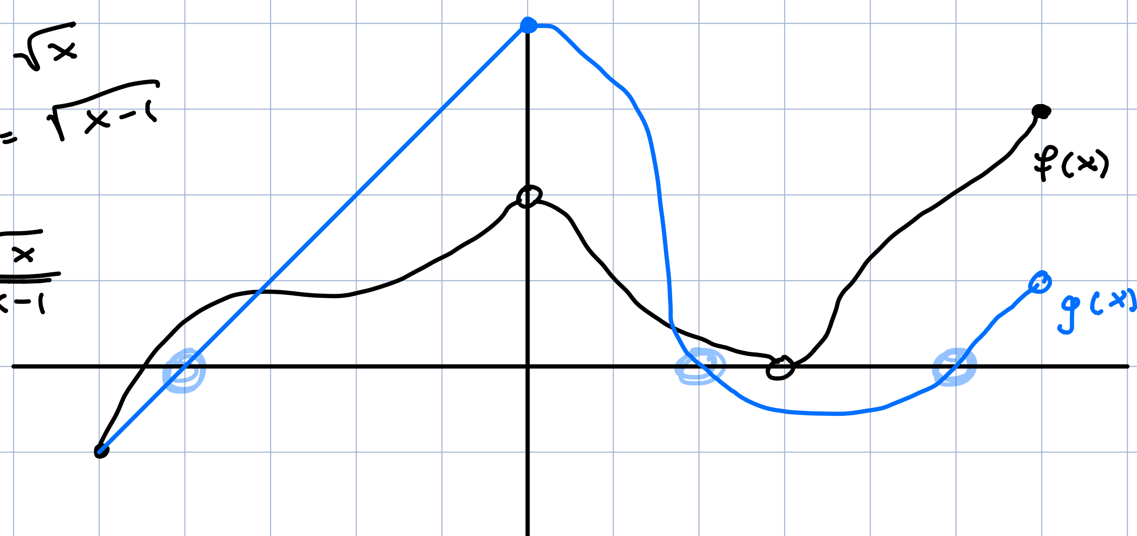


3) What is the domain of f/g where the graphs of f and g are given below

$f(x) = \sqrt{x}$

$g(x) = \sqrt{x-1}$

$f/g = \dfrac{\sqrt{x}}{\sqrt{x-1}}$



$\text{domain}(f/g) = \{x \mid g(x) \neq 0\} \cap \text{domain}(f) \cap \text{domain}(g)$

$\text{domain}(f) = [-5, 0) \cup (0, 3) \cup (3, 6]$

domain(g) = $[-5, 6)$

$\{x \mid g(x) \neq 0\} = [-5, 4) \cup (-4, 2) \cup (2, 5) \cup (5, 6)$

Defn: f, g = fcns, the composition of g w/ f is the fcn $f \circ g(x) = f(g(x))$

Ex: ↳ $f(x) = \sqrt{x}$, $g(x) = x - 3$, then

$f \circ g(x) = \sqrt{x-3}$

$\quad \| $

$f(g(x)) = f(x-3) = \sqrt{x-3}$

↳ $f(x) = x^2$, $g(x) = x^5$

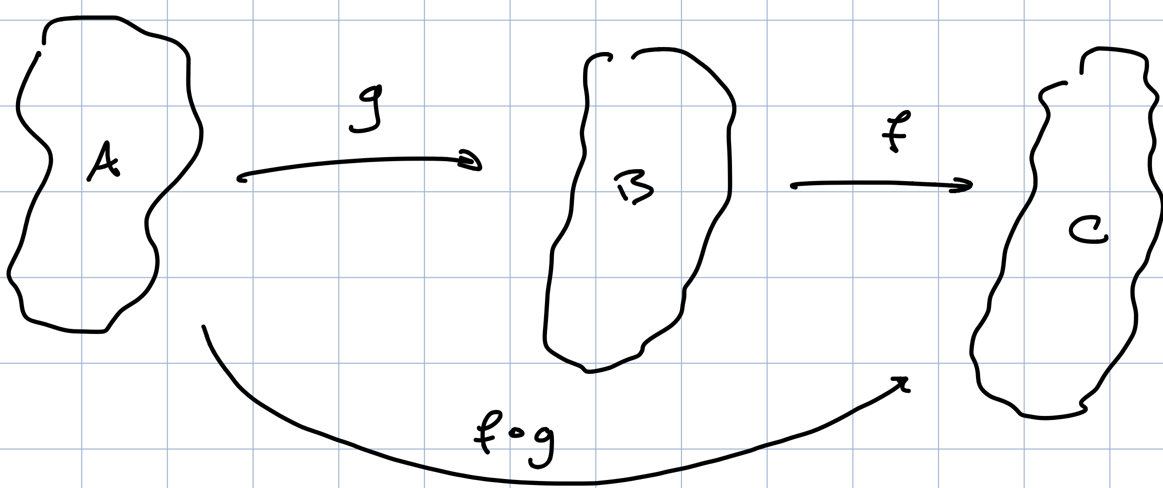
$f \circ g(x) = f(g(x)) = f(x^5) = (x^5)^2 = x^{10}$.

↳ $f(x) = \sqrt{x}$, $g(x) = x^2 - 1$

$f \circ g(x) = \sqrt{x^2 - 1}$

$\quad \|$

$f(g(x)) = f(x^2-1) = \sqrt{x^2-1}$



RmK: domain$(f \circ g)$ = domain$(g) \cap \{x \mid g(x)$ in domain$(f)\}$

↳ $f(g(x))$

↳ $f(x) = x^2$] $f \circ g(x) = \frac{1}{\,}$

$g(x) = \frac{1}{x}$ $\quad f \circ g(x) = x^2$

$\qquad\qquad\qquad\; \parallel$

$\qquad\qquad\quad f(\frac{1}{x})$

↳ $f(x) = \sqrt{x}$, $g(x) = x - 3$

$$\begin{aligned}
&\mathrm{dom}(g) = \mathbb{R} \\
&\{x \mid g(x) \text{ in } \mathrm{dom}(f)\} \\
&\quad \parallel \\
&\{x \mid g(x) \geq 0\} \\
&\quad \parallel \\
&\{x \mid x - 3 \geq 0\} \\
&\quad \parallel \\
&\{x \mid x \geq 3\}
\end{aligned}$$

$$\begin{aligned}
&\mathrm{domain}(f \circ g) = \{x \mid x \geq 3\} \\
&\quad \parallel \\
&\mathrm{domain}(\sqrt{x-3}) = \{x \mid x \geq 3\}.
\end{aligned}$$

Rmk: Decompose fcns as comp. of fcns

↳ $F(x) = \sqrt{x - 3}$

$f(x) = \sqrt{x}$, $g(x) = x - 3$

$\Rightarrow F(x) = f \circ g(x)$.

↳ $F(x) = (x + \pi)^{777}$

$f(x) = x^{777}$, $g(x) = x + \pi$

$f \circ g(x) = f(g(x)) = f(x + \pi) = (x + \pi)^{777}$ ✓

↳ $F(x) = \dfrac{1}{\sqrt{x - 7}} = f \circ g \circ h(x) = f(g(h(x)))$.

$h(x) = x - 7$, $g(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$

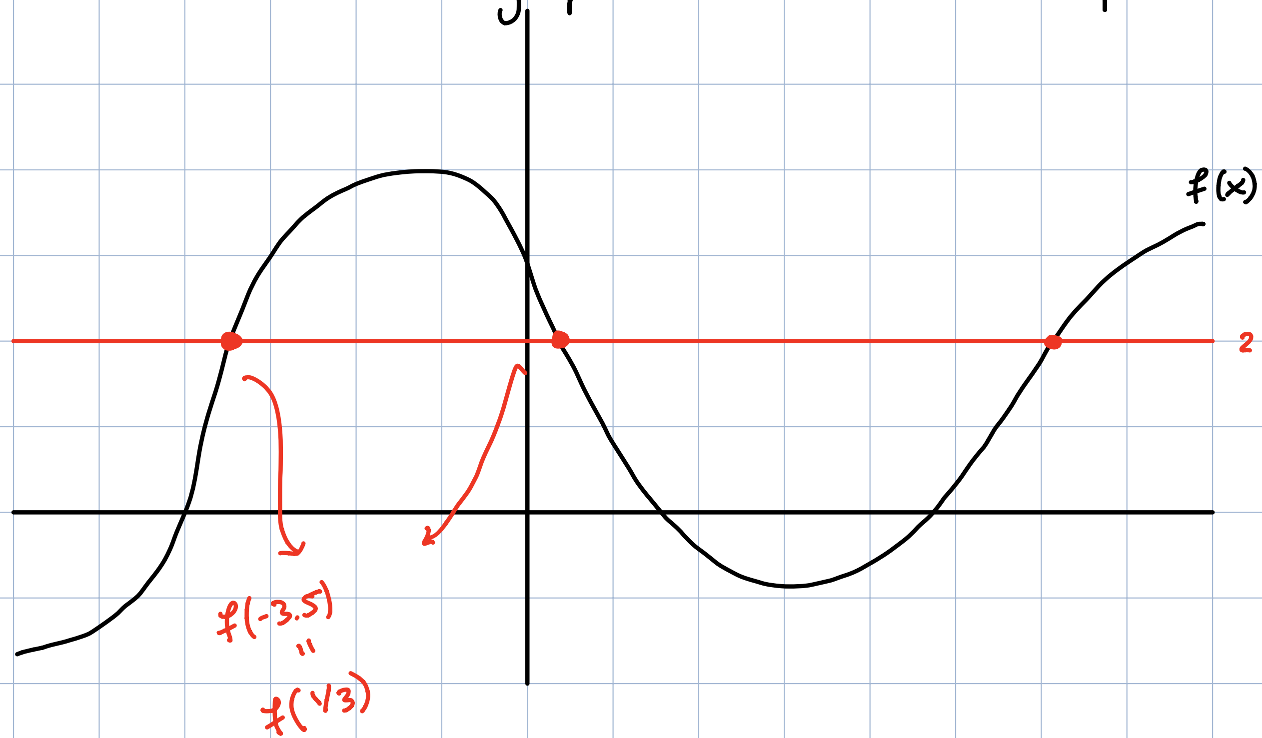
Section 2.8: 1-to-1 fcns and their inverses.

Defn: A fcn f is one-to-one if

$\Leftrightarrow$ $f(x_1) = f(x_2)$, then $x_1 = x_2$

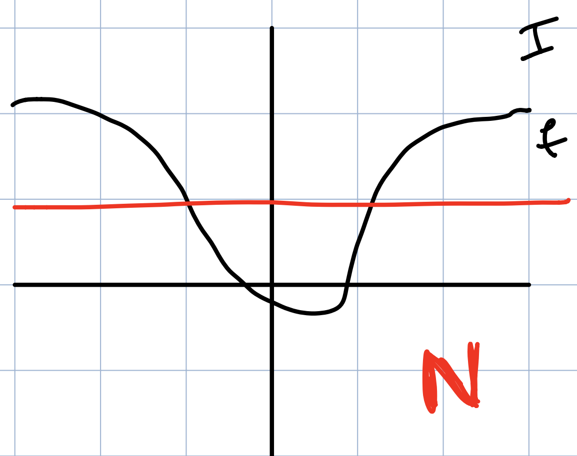
f maps each value x to its own unique value $f(x)$ that is different from $f(x_0)$ for $x_0 \neq x$.

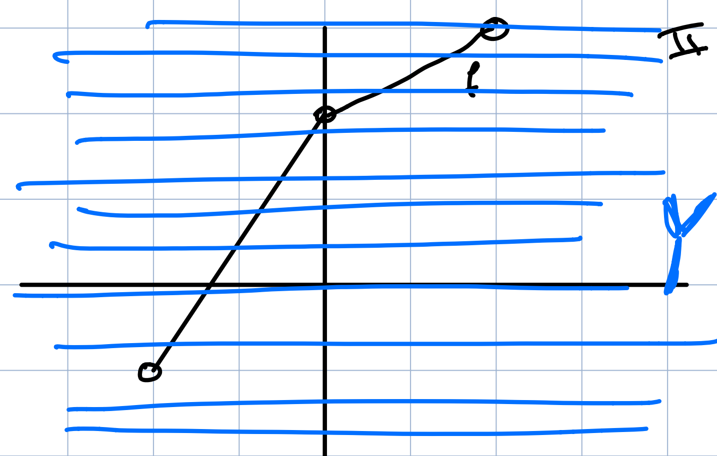
Rmk: Horizontal Line Test: A fcn f is 1-to-1 if every hor. line meets the graph of f at most 1 point.



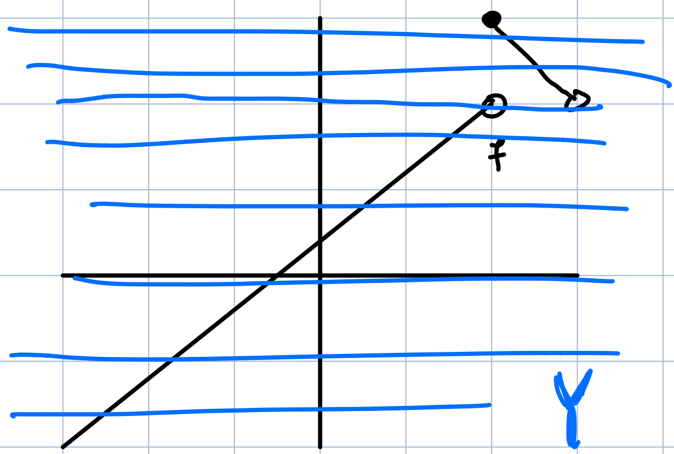
but $-3.5 \neq 1/3$, so f is not 1-to-1.

Ex:

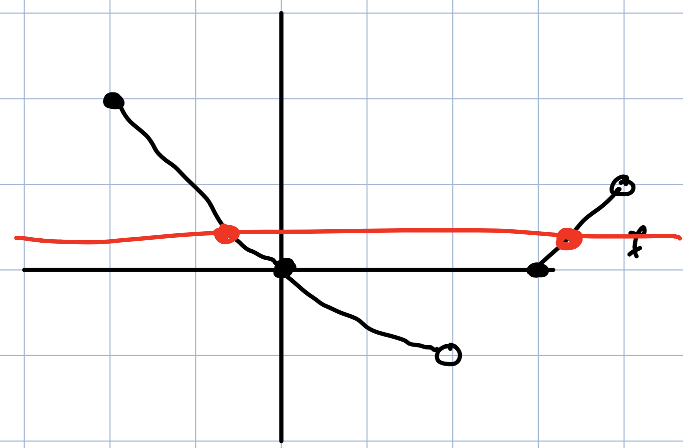




III



IV



Ex: $f(x) = x^3$, $g(x) = x^2$.

a) Is f one-to-one?

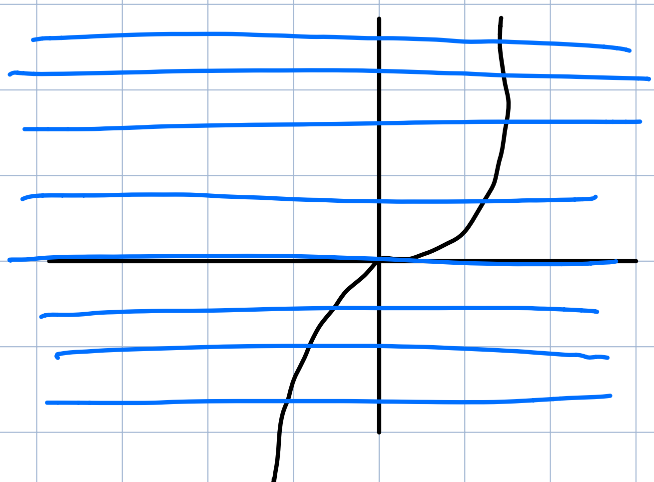
b) Is g " " " ?

↳ $\underset{(x_0)^3}{\underset{\shortparallel}{f(x_0)}} = \underset{(x_1)^3}{\underset{\shortparallel}{f(x_1)}}$, then $x_0 = x_1$

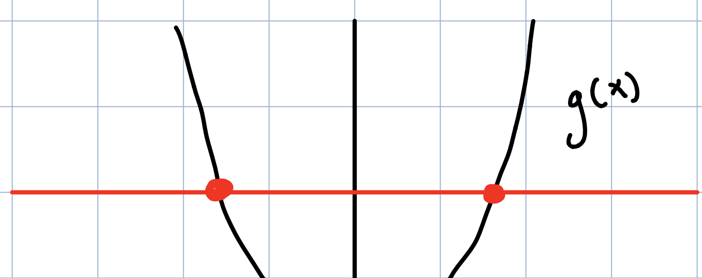
→ $\sqrt[3]{\cdot}$ of both sides, $\underset{x_0}{\underset{\shortparallel}{\sqrt[3]{(x_0)^3}}} = \underset{x_1}{\underset{\shortparallel}{\sqrt[3]{(x_1)^3}}}$

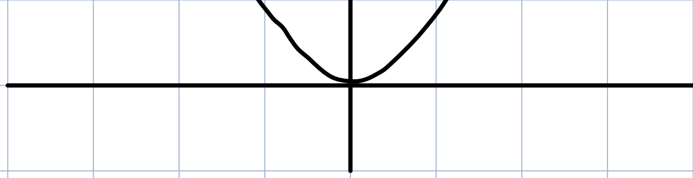
$\Rightarrow x_0 = x_1$

$\Rightarrow f$ is 1-to-1



↳





Spse

$$(x_0)^2 = g(x_0) = g(x_1) = (x_1)^2$$

$\sqrt{\cdot} \Rightarrow \quad x_0 = \pm x_1$

$\Rightarrow$ not nec. equal.

Ex: $f(x) = 3x + 2$, is it one-to-one?

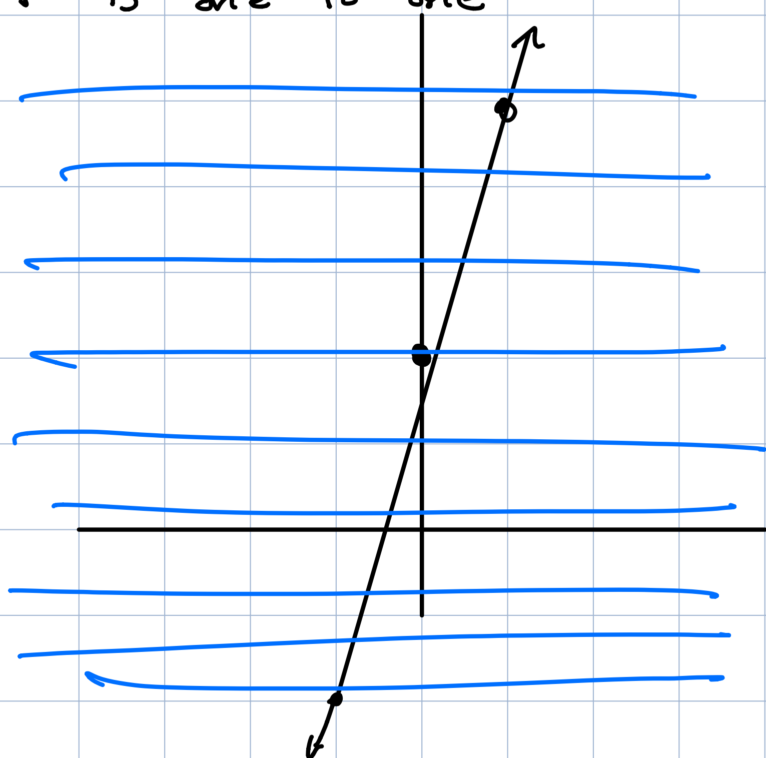
Alg, we assume $\underline{f(x_0) = f(x_1)}$ and need to show $x_0 = x_1$

$$3x_0 + 2 = f(x_0) = f(x_1) = 3x_1 + 2$$

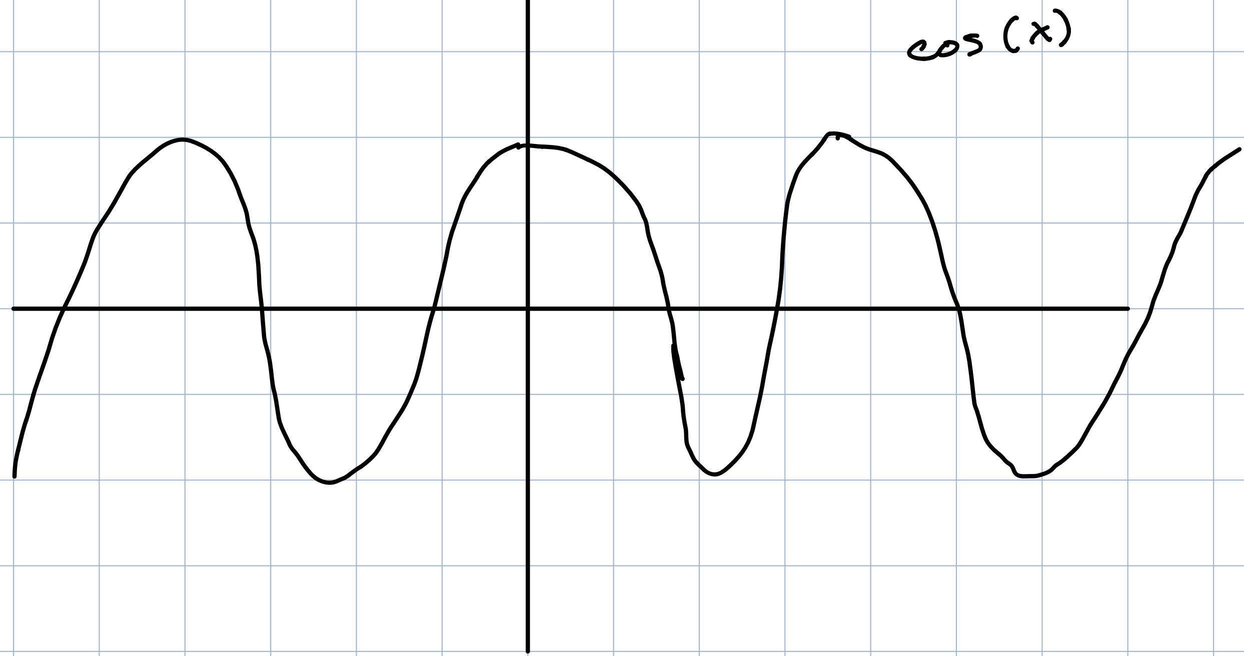
$\Rightarrow \quad 3x_0 = 3x_1$

$\Rightarrow \quad x_0 = x_1$

$\Rightarrow f$ is one-to-one



✓



Ex: $f(x) = x^4 - 5$

Spse $f(x_0) = f(x_1)$

if $x_0 = x_1$, then f is 1-to-1

" not, then " " not "

$(x_0)^4 - 5 = f(x_0) = f(x_1) = (x_1)^4 - 5$

$\Rightarrow x_0^4 = x_1^4$

$\Rightarrow \sqrt[4]{\cdot} \rightsquigarrow x_0 = \pm x_1$

$\left.\begin{matrix} x_0 = 2 \\ x_1 = -2 \end{matrix}\right\} \Rightarrow \begin{matrix} 2^4 & = & (-2)^4 & & \\ \shortparallel & & \shortparallel & & \\ 16 & & (-1)^4 16 & = & 16. \end{matrix}$

$\Rightarrow f$ is not 1-to-1

Ex: $f(x) = x^2 - 2x + 3$, is 1-to-1 or not?

$f(x_0) = f(x_1)$

$x_0^2 - 2x_0 + 3 = x_1^2 - 2x_1 + 3$

$\Rightarrow x_0^2 - 2x_0 = x_1^2 - 2x_1$

$\Rightarrow 1 x_0^2 - 2x_0 + (2x_1 - x_1^2) = 0$

$x_0 = \pm x_1$

$a x_0^2 + b x_0 + c = 0$

Quad form

$$\Rightarrow \quad x_0 = \frac{+2 \pm \sqrt{4 - 4(2x_1 - x_1^2)}}{2}$$

$\Rightarrow$ x_0 can be mult things and have exp hold.

Ex: $f(x) = x^2 - 1$, is 1-to-1 ?

↳ Find counter exp $(x_0, x_1$ w/ $f(x_0) = f(x_1)$ $x_0 \neq x_1)$.

↳ zeros of f

$f(1) = 0$

$f(-1) = 0$