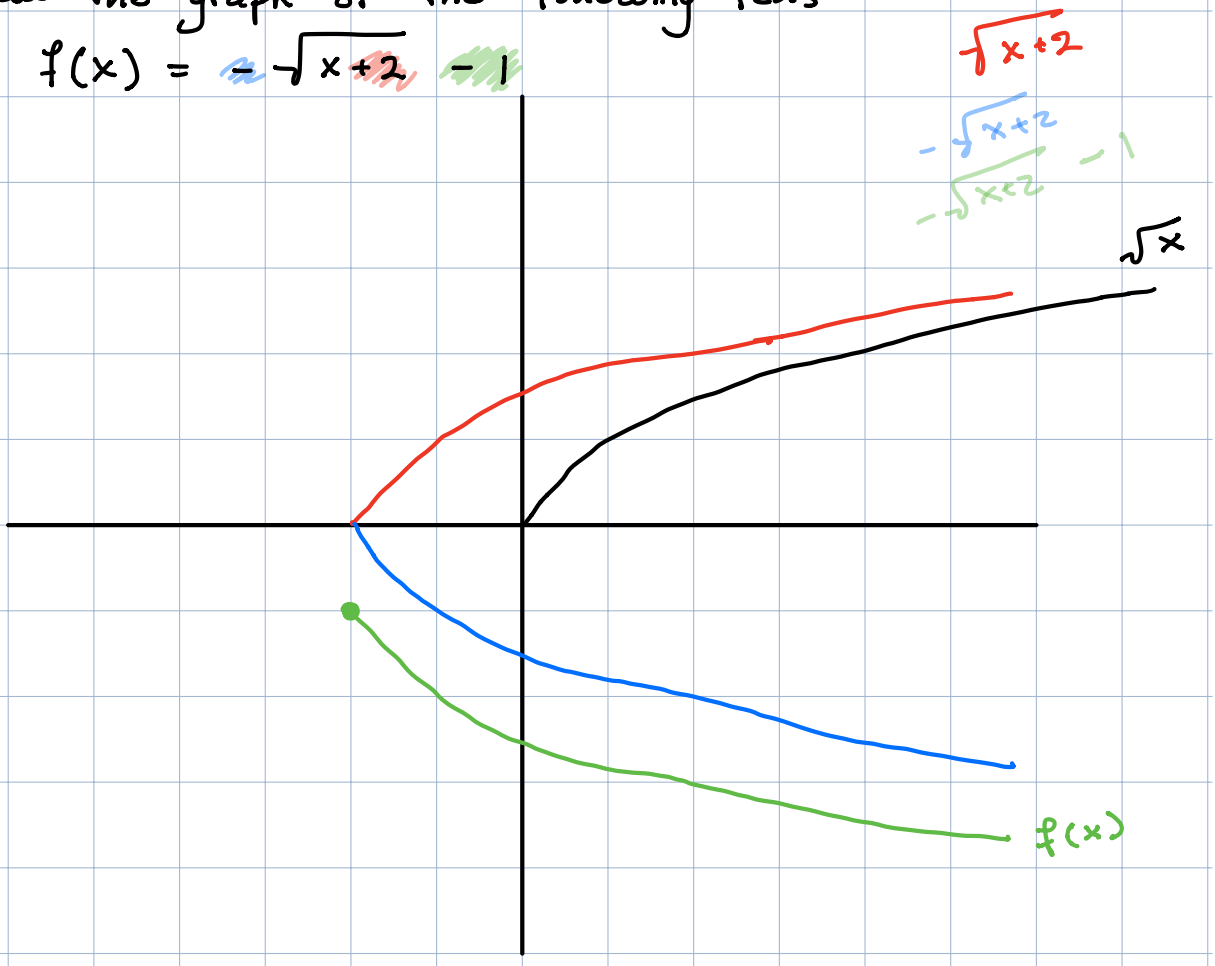
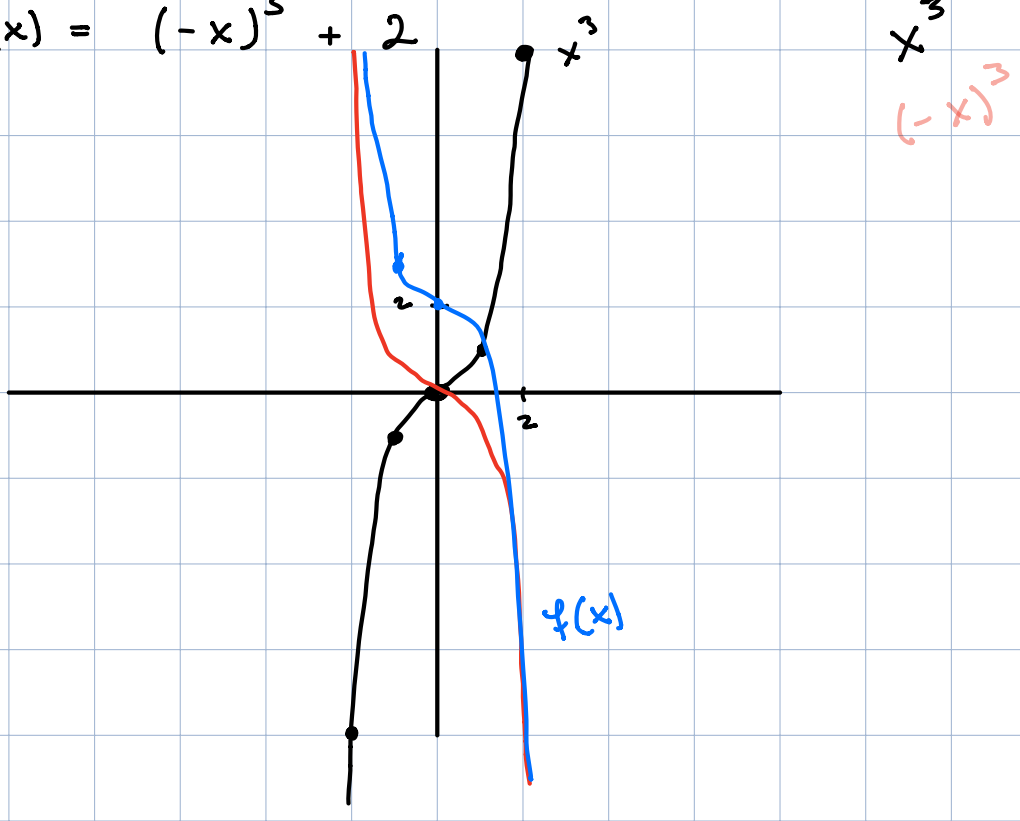
Lecture # 8

Warm-up: 1) Draw the graph of the following fcns

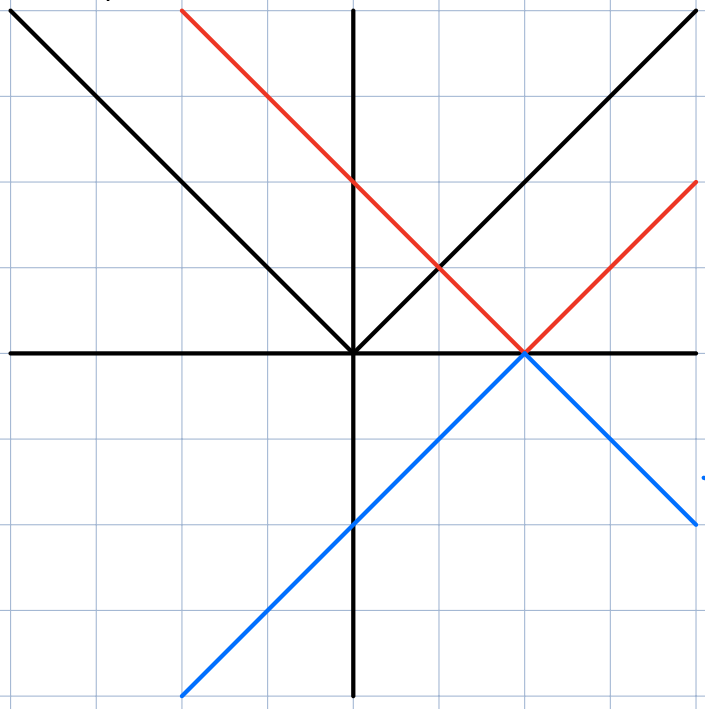
a) $f(x) = -\sqrt{x+2} - 1$



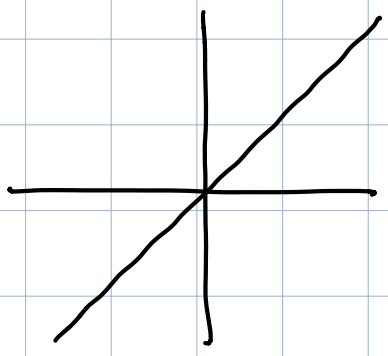
b) $f(x) = (-x)^3 + 2$



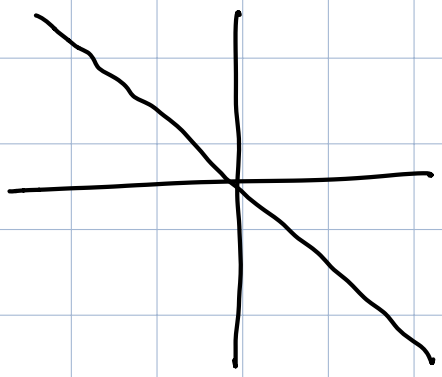
c) $f(x) = -|x-2|$



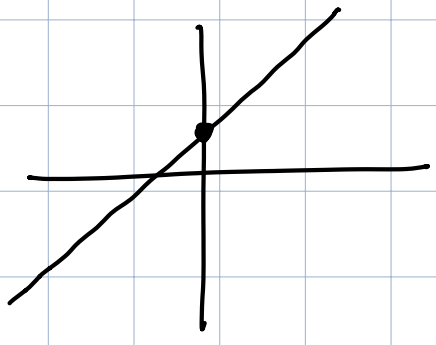
Rmk: Think about what happens to $f(x) = x$. $(3x+2)$



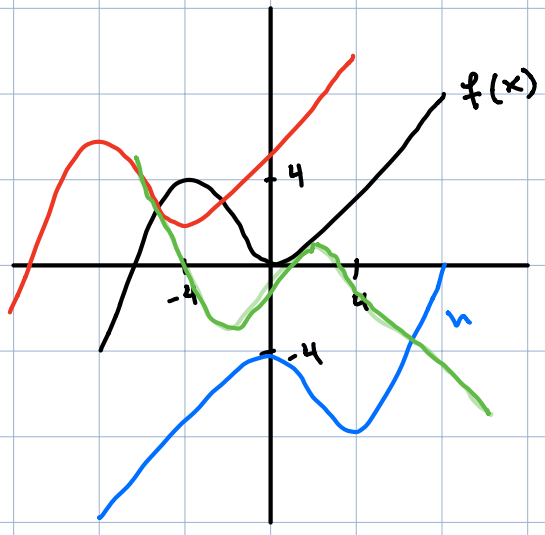
$f(-x) = -x$



$f(x+1) = x+1$



2) Consider the fcn f w/ graph



Draw the graphs of the following fcns

a) $g = f(x+4) + 2$

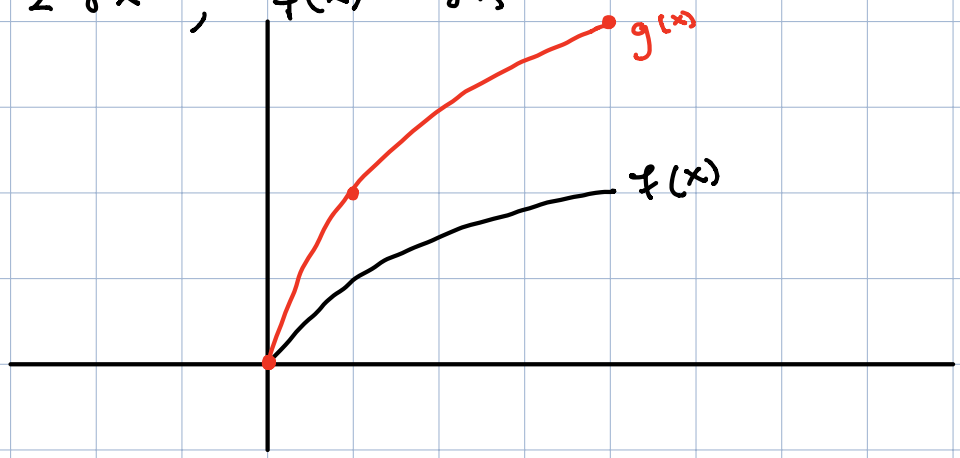
b) $h = -f(-x) - 1$

c) $K = -f(x-2) + 1$

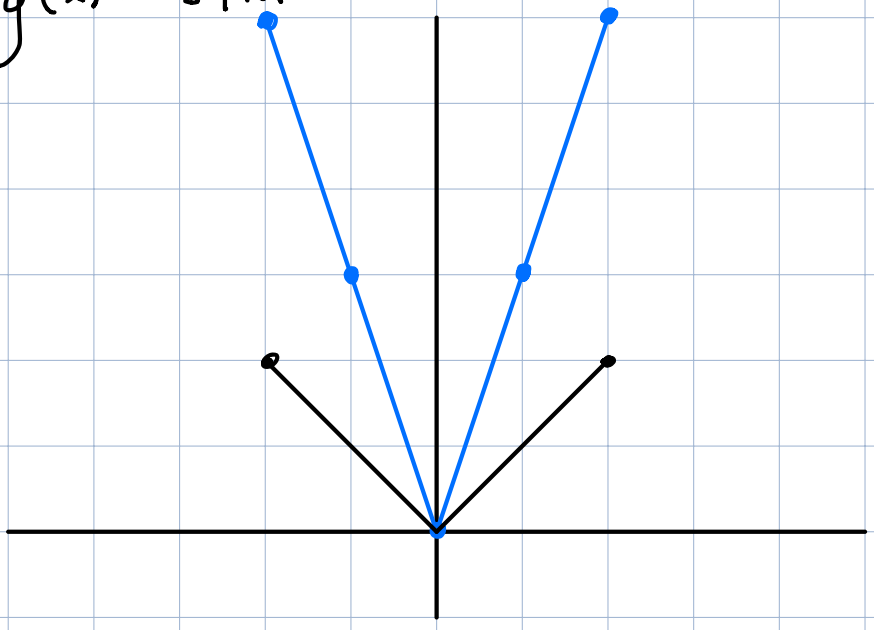
RmK: Vertical stretching

$y = c \cdot f(x)$ $(c > 0)$ $\rightsquigarrow$ stretch the graph vertically by a factor of c.

$\hookrightarrow$ $g(x) = 2\sqrt{x}$, $f(x) = \sqrt{x}$



↳ $g(x) = 3|x|$

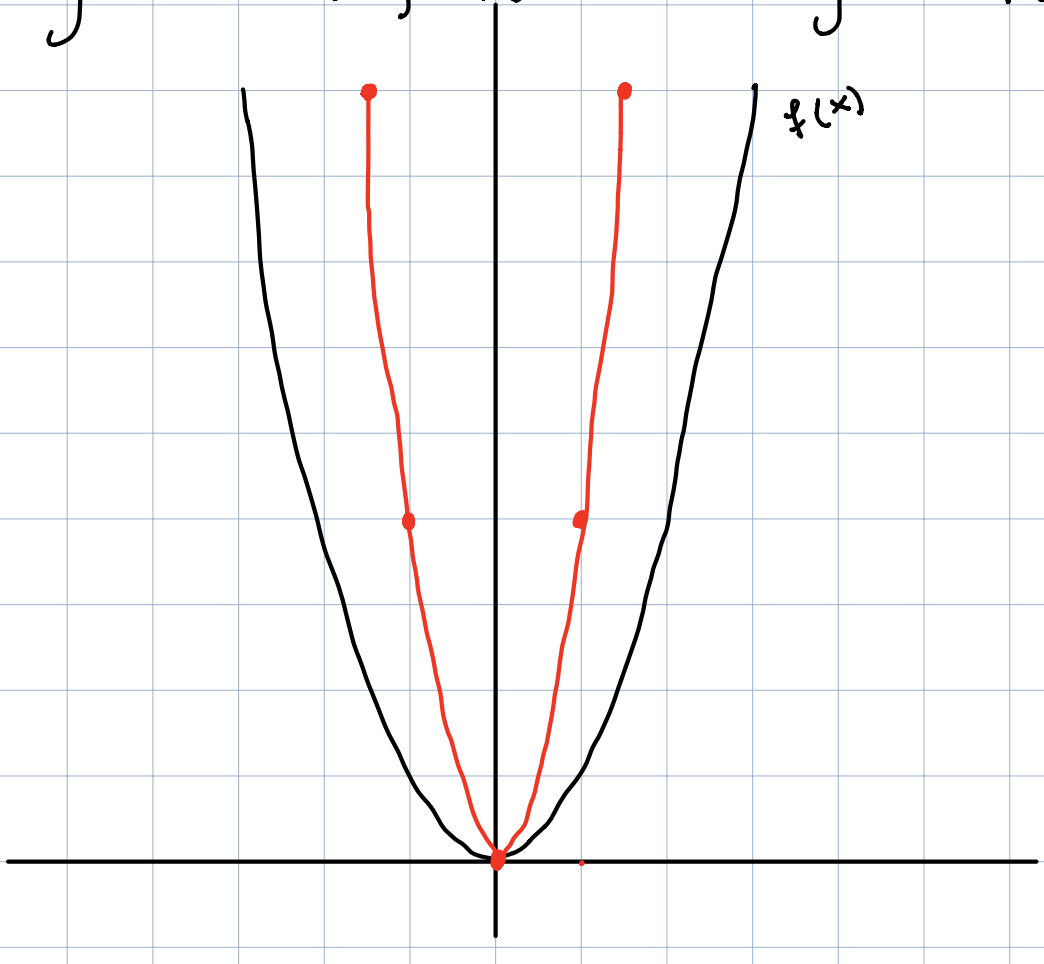


Rmk: Horizontal Stretching

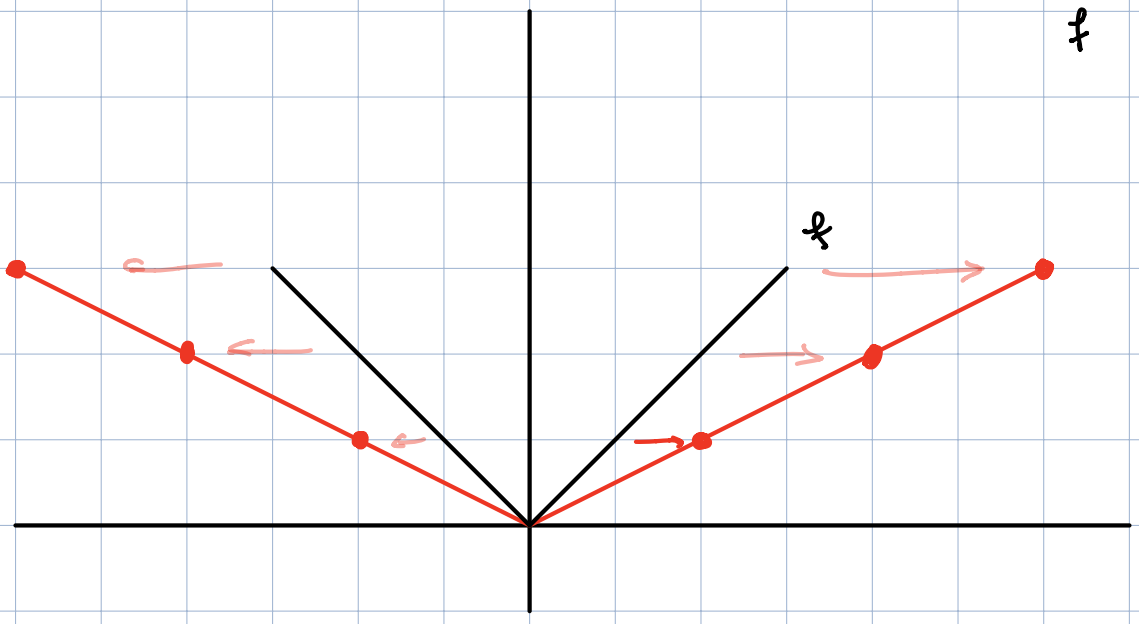
$y = f(c \cdot x)$, $(c > 0)$ ⇝ stretch horizontally (x-dir).

by a ~~factor~~ of $1/c$

↳ $g(x) = (2x)^2$, $f(x) = x^2$ ⇝ $g(x) = f(2x)$.



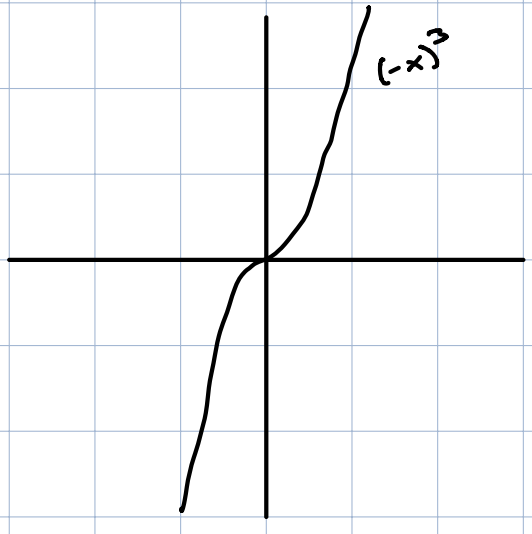
↪ $g(x) = \left|\frac{x}{2}\right|$; $f(x) = |x|$, $g(x) = f(x/2)$



Defn: A fcn $f(x)$ is <u>odd</u> if $f(-x) = -f(x)$

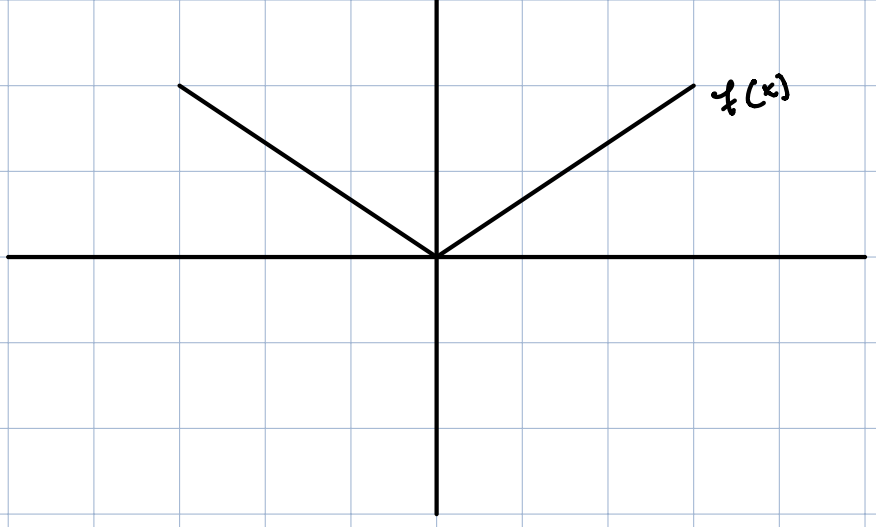
↪ Ex: $f(x) = x^3$

$f(-x) = (-x)^3 = (-1)^3 \cdot x^3 = -(x^3) = -f(x)$



↪ f is odd if and only if its graph is sym wrt the origin.

Quest:



is $f(x)$ an odd fcn?

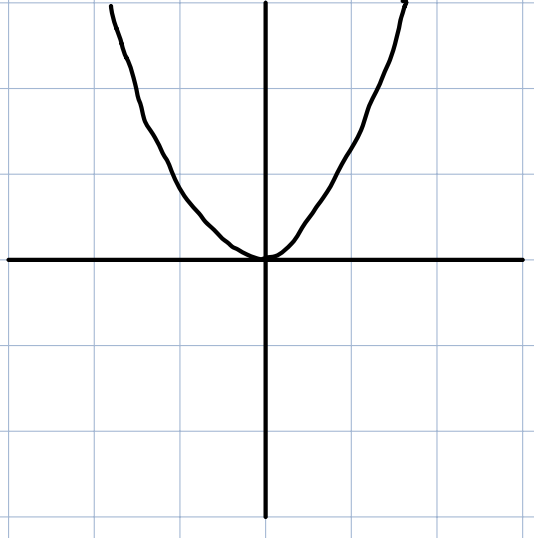
↳ $f(x) = \left|\frac{2}{3}x\right|$

$$f(-x) = \left|-\frac{2}{3}x\right| = \left|\frac{2}{3}x\right| \neq -f(x) = -\left|\frac{2}{3}x\right| \quad \text{for } x \neq 0$$

Defn: A fcn $f(x)$ is <u>even</u> if $f(-x) = f(x)$

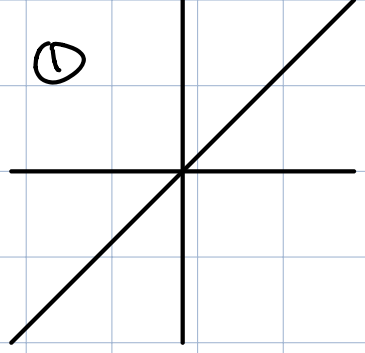
↳ $f(x) = x^2$

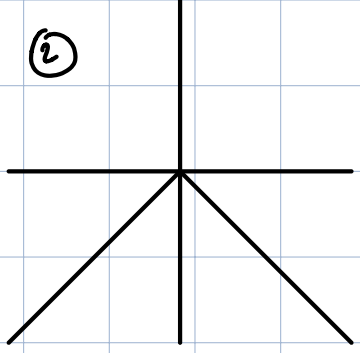
$$f(-x) = (-x)^2 = (-1)^2 x^2 = x^2 = f(x)$$

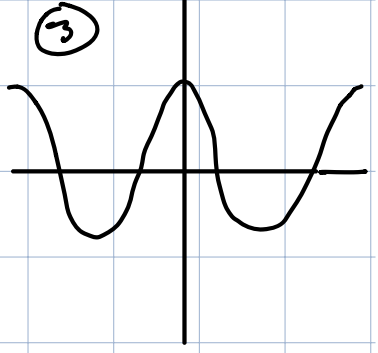


↳ f is even when its graph is sym with respect to the y-axis.

Quest:







Even, odd, both, neither?

① not even; yes odd.

② even

③ even

Ex: 1) $f(x) = x^5 - x$

is ✓ $f(-x) = -f(x)$ for odd

$f(-x) = f(x)$ for even.

↳ $f(-x) = (-x)^5 - (-x)$

$= (-1)^5 x^5 + x$

$= -x^5 + x$

$= -(x^5 - x)$

$= -f(x)$

$\Rightarrow f$ is odd.

To see that it is not even, Suff. to test some value and show above req. fails.

$f(2) = 2^5 - 2 = 32 - 2 = 30$

$f(-2) = (-2)^5 - (-2) = -32 + 2 = -30$

2) $f(x) = \sqrt{x^2 - 1}$, $f(-x) = \sqrt{(-x)^2 - 1}$

f is even. $= \sqrt{x^2 - 1}$

$= f(x)$.

Section 2.7: Combining Fcn

Rmk: Let f, g be two fcns.

- $(f+g)(x) = f(x) + g(x)$.
  - $f(x) = x^2$, $g(x) = x^3 \Rightarrow (f+g)(x) = x^2 + x^3$.
- $(f-g)(x) = f(x) - g(x)$
  - " , " $\Rightarrow (f-g)(x) = x^2 - x^3$.
- $(f \cdot g)(x) = f(x) \cdot g(x)$
  - $f(x) = x^2$, $g(x) = x^3 \Rightarrow (f \cdot g)(x) = x^2 \cdot x^3 = x^5$
- $(f/g)(x) = f(x)/g(x)$ (require $g(x) \neq 0$)
  - " , " $\Rightarrow (f/g)(x) = x^2/x^3 = \frac{1}{x}$

Rmk:

- Domain of $f+g$ = $\text{dom}(f) \cap \text{dom}(g)$
  - $f = \frac{1}{x}$, $g(x) = \frac{1}{x-1}$

    $(f+g)(x) = \frac{1}{x} + \frac{1}{x-1}$
- " " $f-g$ = " $\cap$ "
- " " $f \cdot g$ = " $\cap$ "
- " " f/g = $\text{dom}(f) \cap \text{dom}(g) \cap \{x \mid g(x) \neq 0\}$.

Ex: $f(x) = \frac{1}{x-3}$, $g(x) = \sqrt{x}$

Question: a) What is the dom. of $f-g$?

b) " " " " " f/g?

c) " " " " " g/f?

Answers: a) $\{x \mid x \geq 0 \text{ and } x \neq 3\}$ $\cap$ <--> and

b) $\{x \mid x \geq 0 \text{ and } x \neq 3 \text{ and } x \neq 0\}$ $\cup$ <--> or

$= \{x \mid x > 0 \text{ and } x \neq 3\}$.

c) $\{x \mid x \geq 0 \text{ and } x \neq 3\}$

$x/2$

$f(x) = x$

$g(x) = 2$

$\Big\}$ $f/g = \frac{x}{2}$