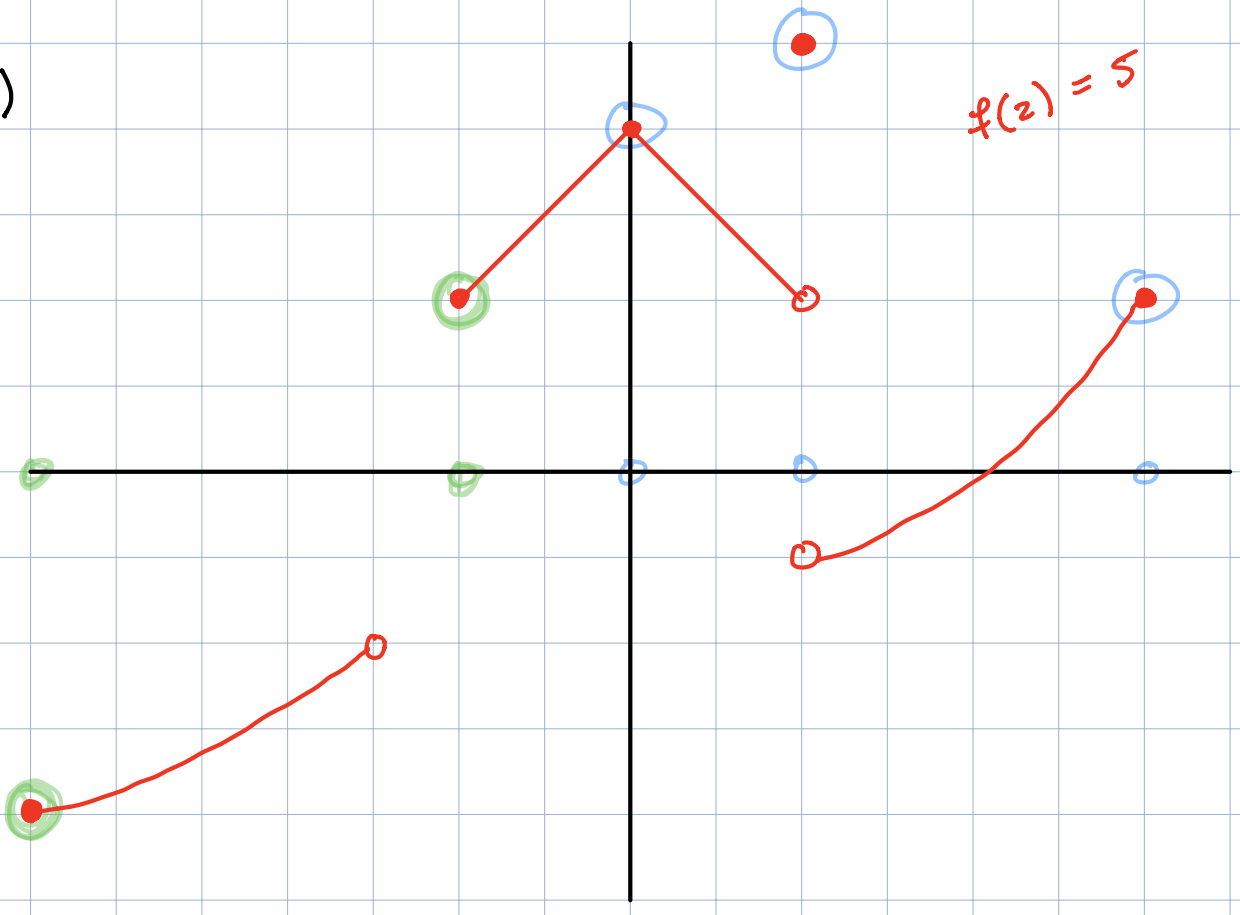
Lecture # 7

Warm-up: 1)



Consider the function whose graph is given above. What is

a) its domain $[-7,-3) \cup [-2,6]$

b) its range $[-4,-2) \cup (-1,4] \cup 5$.

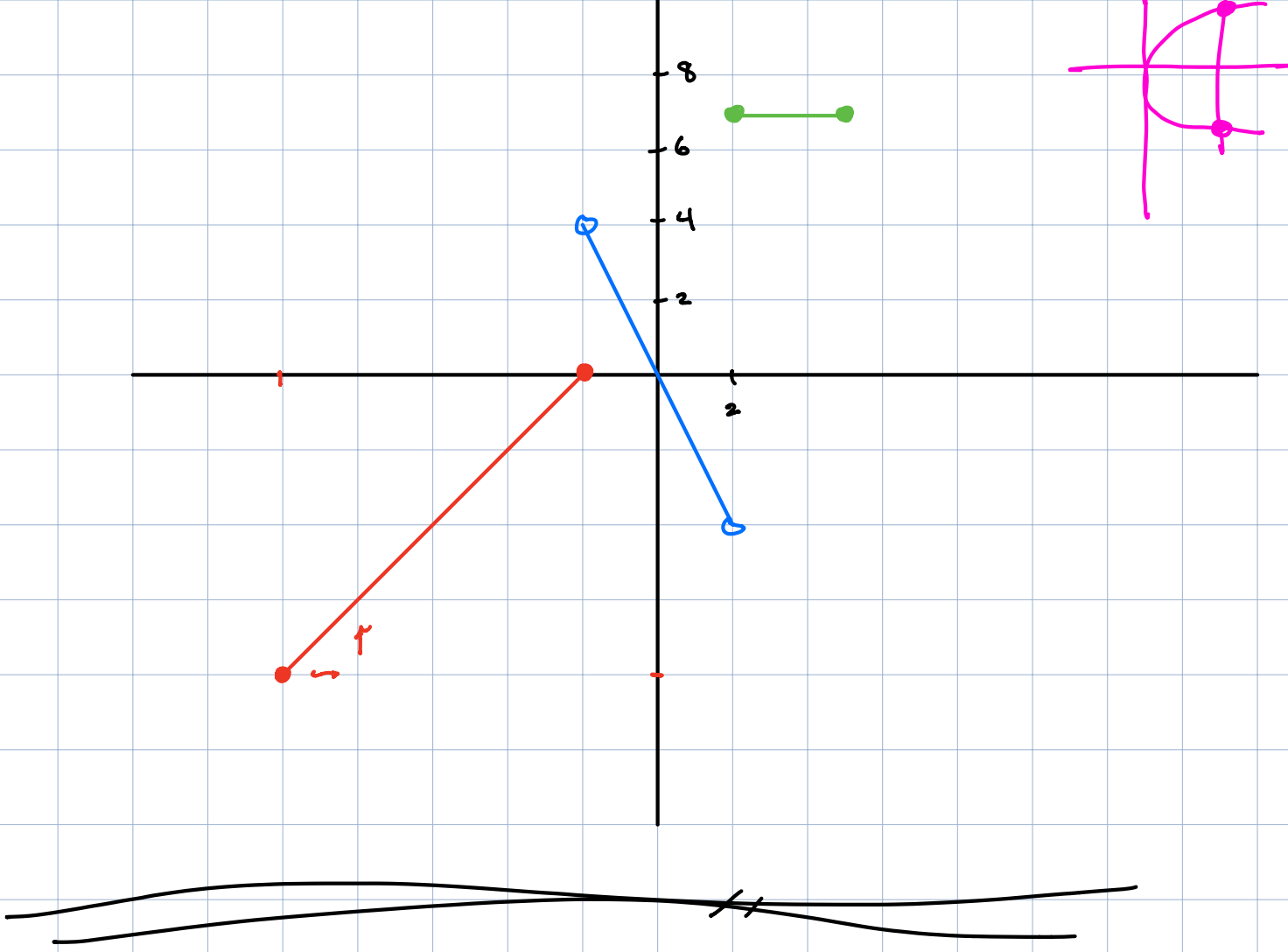
c) values of x w/ loc. max

d) " " " " loc. min.

2) Graph the following function

$$f(x) = \begin{cases} x-3, & -5 \le x \le -2 \\ -2x, & -2 < x < 2 \\ 7, & 2 \le x \le 5 \end{cases}$$

$y^2 = x$



Section 2.4

Rmk: Average speed (rate of change) = dist. travels / time travelled.

If f is a fcn w/ $f(t)$ = dist car travels, then

$$\left(\begin{array}{c}\text{aver rate of chang}\\ \text{of } f \text{ from}\\ a = \text{time 1},\ b = \text{time 2}\end{array}\right) = \frac{f(b) - f(a)}{b - a}$$

Defn: The ave. rate of change of a fcn f from a to b is $\dfrac{f(b) - f(a)}{b - a} = \dfrac{\text{net change } f \text{ from } a \text{ to } b}{b - a}$

Ex: $f(x) = x^2$ from 0 to 2.

↳ $b = 2, \; a = 0$

$$\frac{f(2) - f(0)}{2 - 0} = \frac{2^2 - 0^2}{2} = \frac{4}{2} = 2$$

$$f(x) = \frac{1}{x^2 + 1} \quad \text{from } -1 \text{ to } 3$$

↳ $$\frac{f(3) - f(-1)}{3 - (-1)} = \frac{\frac{1}{10} - \frac{1}{2}}{4}$$

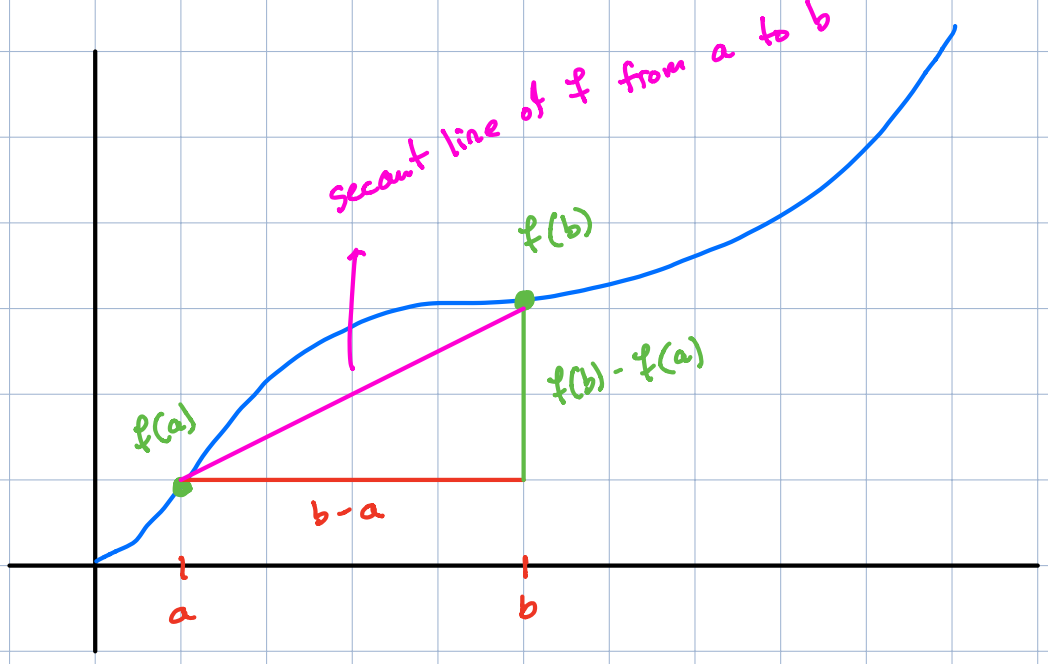
$$= \frac{\frac{1}{10} - \frac{5}{10}}{4}$$

$$= \frac{1}{40} - \frac{5}{40}$$

$$= \frac{-4}{40}$$

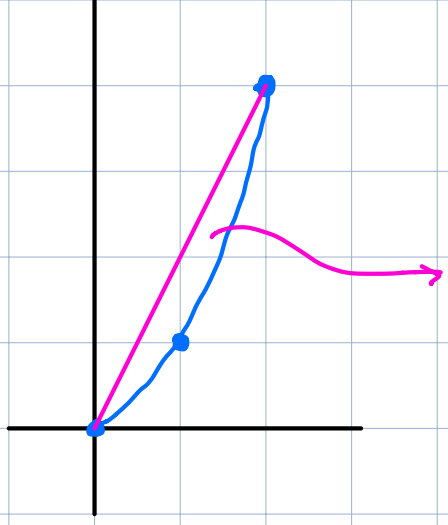
$$= \frac{-1}{10}$$

Picture:



Slope of the secant line is the ave. rate of change of f from a to b.

Ex 8



$f(x) = x^2$

slope $= \frac{4}{2} = 2$, ü

Section 2.6

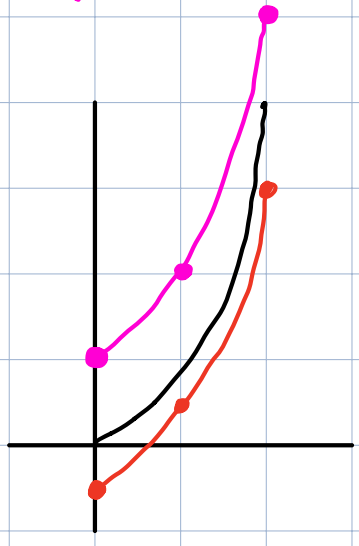
Rmk 8 Vertical Shifting

Graph of the eqn $y = f(x) \pm c$ ($c \geq 0$), then the graph is the graph of $y = f(x)$ shifted up/down by c.

↪ $f(x) = x^2$

$f(x) = x^2 - \frac{1}{2}$

$f(x) = x^2 + 1$



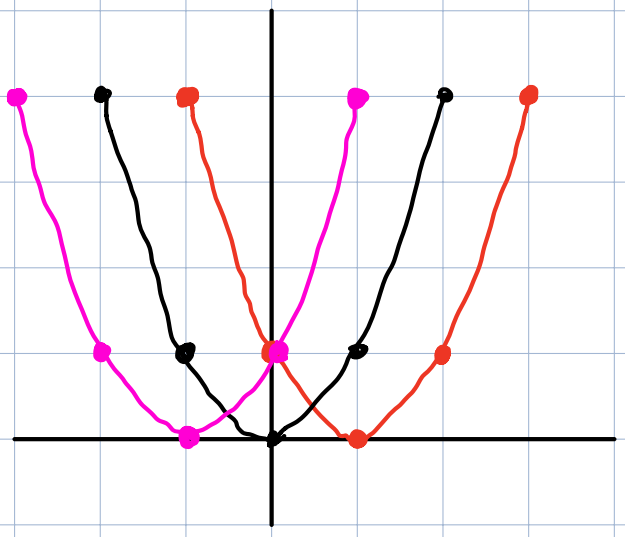
Rmk: Horizontal shifting.

The graph of $y = f(x \pm c)$ is the graph of $f(x)$ shifted hor. to the left / right by c.

↳ $f(x) = x^2$

$f(x-1) = (x-1)^2$

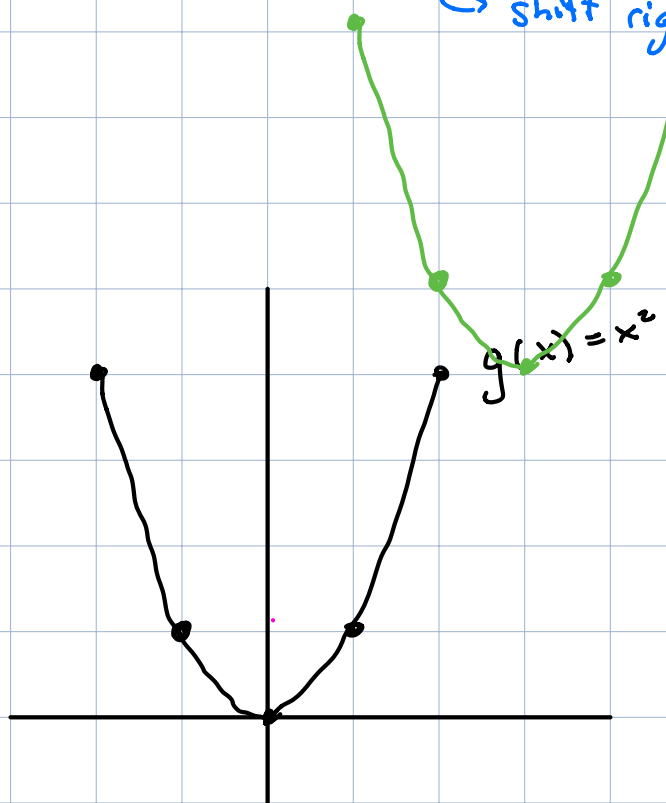
$f(x+1) = (x+1)^2$



Ex: $f(x) = (x-3)^2 + 4 = g(x-3) + 4$ ⇝ shift up by 4

$g(x) = x^2$

↳ shift right by 3

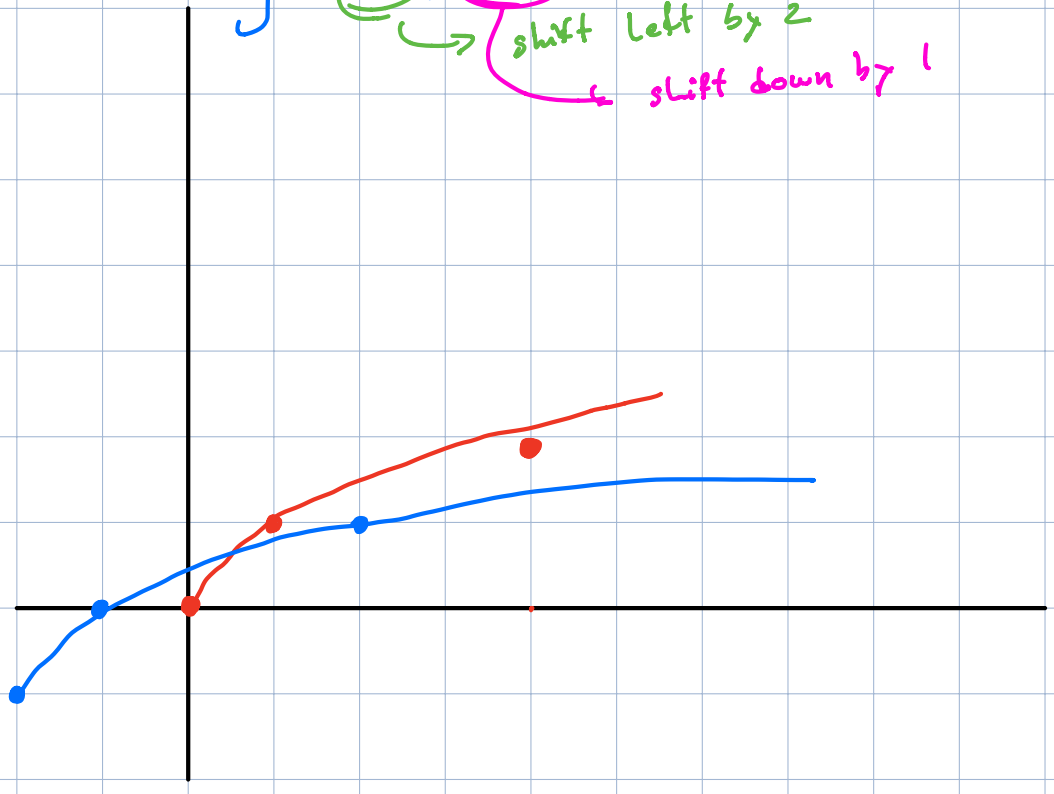


Ex: $f(x) = \sqrt{x+2} - 1$, $g(x) = \sqrt{x}$

$= g(x+2) - 1$

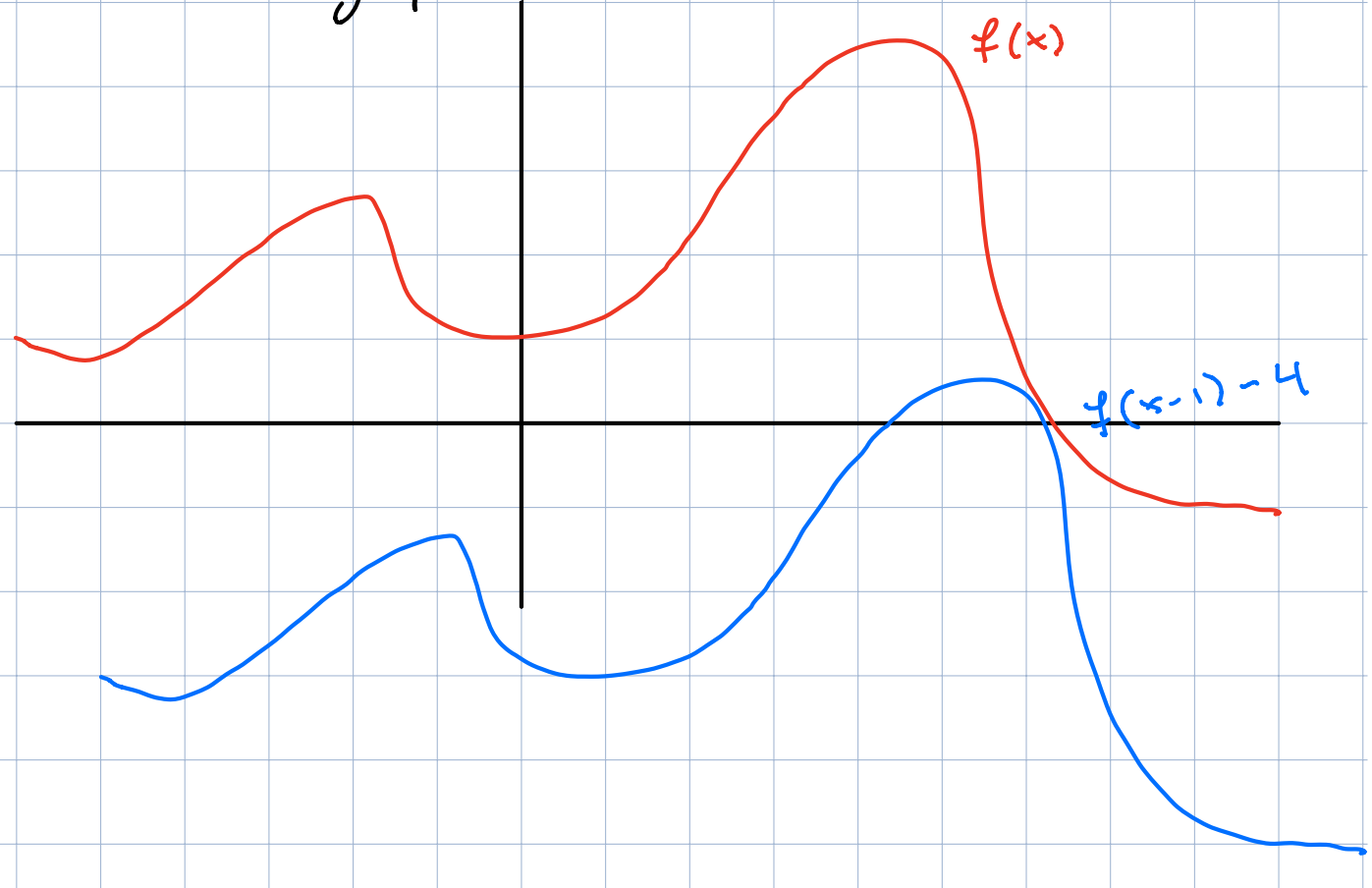
shift left by 2

shift down by 1



Ex: $f(x)$ have the following graph

What is the graph of $f(x-1) - 4$



Rmk: Reflecting graphs

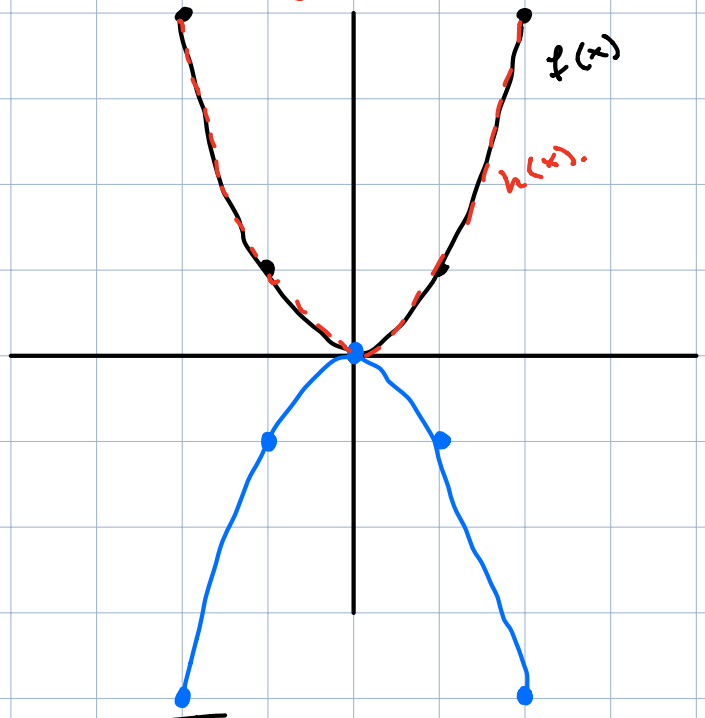
$y = -f(x)$ => graph of $f(x)$ reflected across the x-axis

$y = f(-x)$ => " " " " " y-axis.

↳ $f(x) = x^2$

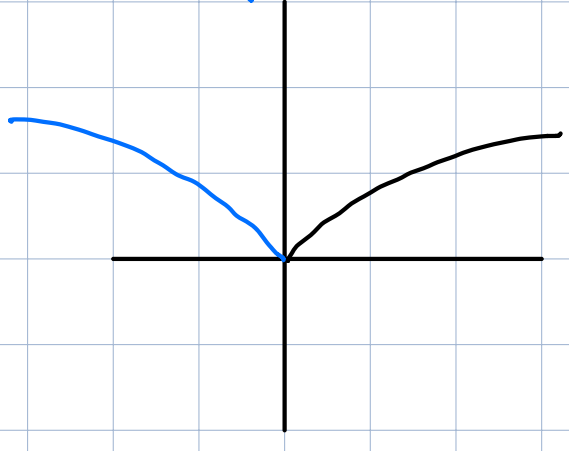
$g(x) = -f(x) = -x^2$

$h(x) = f(-x) = (-x)^2 = x^2$ ← $(-1)^2(x)^2 = 1 \cdot x^2$



↳ $f(x) = \sqrt{x}$

$g(x) = \sqrt{-x} = f(-x)$.



Ex: $f(x) = (-x+1)^2 - 2$

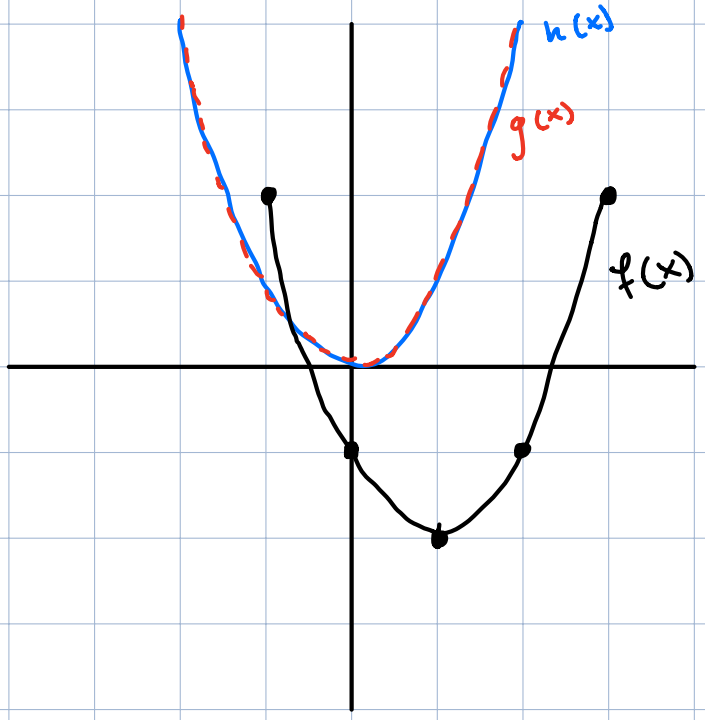
$h(x) = x^2$

$g(x) = h(-x)$

So $f(x) = g(x-1) - 2$

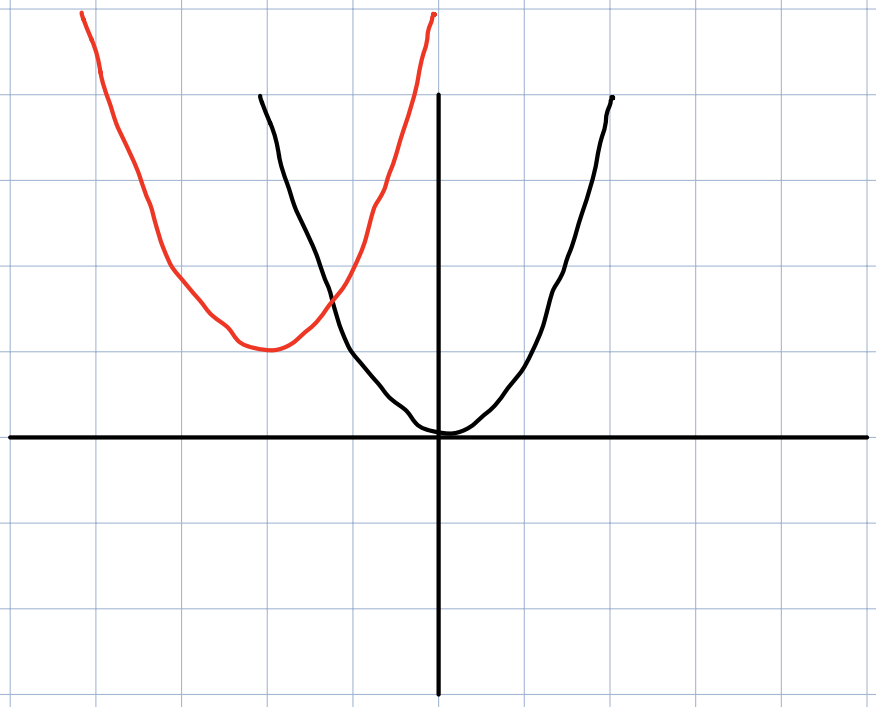
We obtain the graph of g from the graph of h by reflecting across the y-axis. (so it doesn't change)

We obtain the graph of f from the graph of g by shifting to the right by 1 and down by 2



Ex: $f(x) = 1 + (x+2)^2$

$g(x) = x^2$



Ex: Additional example b/c of my mess up.

$$f(x) = \sqrt{-x+2} - 1$$

We would like to obtain the graph of $f(x)$ from the graph of

$$h(x) = \sqrt{x}.$$

Notice that

$$\begin{aligned} f(x) &= h(-x+2) - 1 \\ &= h(-(x-2)) - 1 \end{aligned}$$

Let $g(x) = h(-x)$, then

$$f(x) = g(x-2) - 1$$

So we get the graph of f from the graph of g which we get from the graph of h