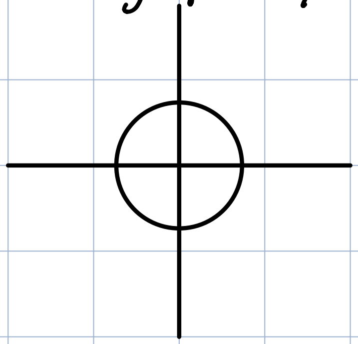
Lecture #6

Defn: (Correction) A graph in the plane is sym wrt the origin if (x, y) is in the graph

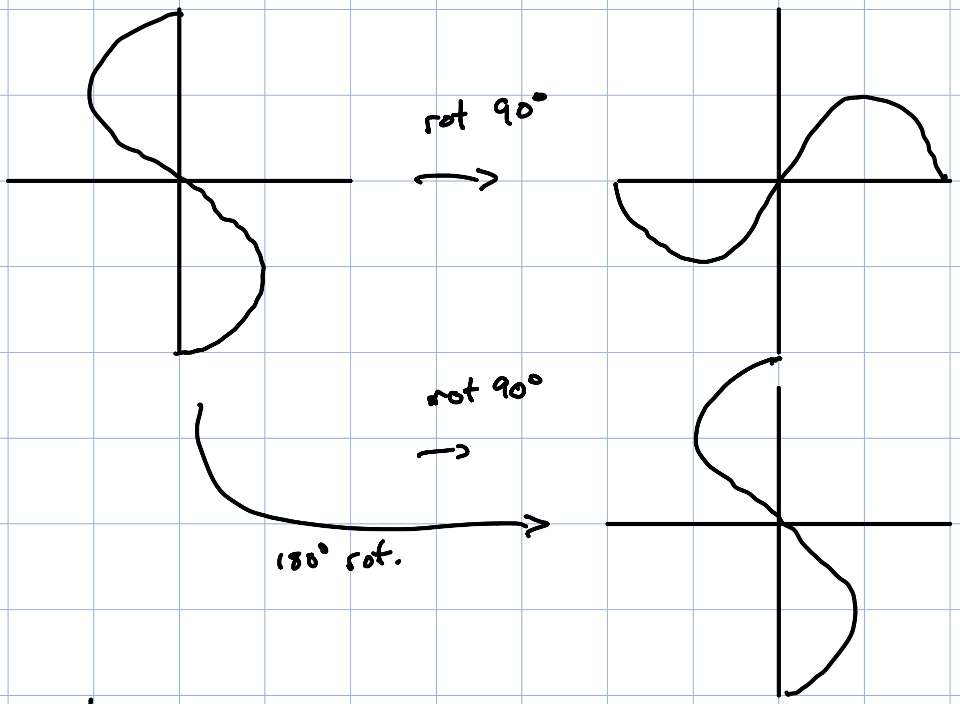
$\Rightarrow (-x, -y)$ " " " "

↪ Rotating the graph by 180° takes it into/onto itself.

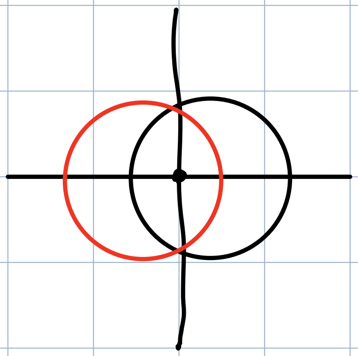
↪ Ex:



sym wrt origin



↪



$\Rightarrow$ not sym wrt origin.

" " " y-axis.

x-axis.

RmK: $x^2 + ax + y^2 + by = c$

↳ It could be a circle $(r > 0)$

↳ " " " " point $(r = 0)$

↳ " " " nothing $(r < 0)$.

Ex: $x^2 - 4x + y^2 = 10$

↳ Write it as $(x - x_0)^2 + (y - y_0)^2 = r^2$

$$x^2 - 4x + 4 + y^2 = 14$$

$$(x-2)^2 + y^2 = 14$$

↳ circle centered at $(2, 0)$ w/ radius $\sqrt{14}$

Ex: $\underline{(x-777)}^2 + \underline{(y-888)}^2 = 0$

$\longrightarrow$ both have to be zero.

$(777, 888)$ is this the graph

Ex: $\underset{\geq 0}{\underline{(x-7)^2}} + \underset{\geq 0}{(y-42)^2} = -43$

$\Rightarrow$ LHS ≥ 0, but RHS < 0

$\Rightarrow$ No solns

$\Rightarrow$ graph is nothing!

RmK: 1.10 = graphing linear eqns (may want to review)

Section 2.1

Defn: A function is a rule that assigns each number in a set A to exactly one # in a set B.

↳ Write $f: A \longrightarrow B$

↳ Ex: f(time) = ave. temperature

f(date) = # of people alive w/ 11 fingers.

f(time worked) = amount of money made.

↳ fcn takes inputs to outputs

Ex: $f(x) = x$ => f takes the # x to the # x.

$f(2) = 2$

$f(3) = 3$

$f(542) = 542$

Ex: $f(x) = x^2$ => " " " " " " " x^2

$f(0) = 0^2 = 0$

$f(2) = 2^2 = 4$

Ex: $f(x) = 5$ => f " " # x to the # 5

$f(7) = 5$

$f(55) = 5$

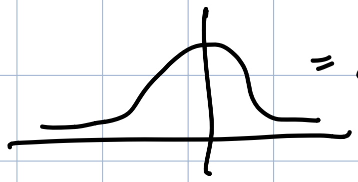
$f(66) = 5$

Ex: $f(x) = \sqrt{x} + x - 7$

$f(0) = \sqrt{0} + 0 - 7 = -7$

$f(9) = \sqrt{9} + 9 - 7 = 3 + 9 - 7 = 5$

Ex: $f(x) = \dfrac{1}{1 + x^2}$ ⟶



= graph of f.

f(−1000) = is very small

f(1000) = " " big

f between −1, 1 is a bump.

Defn: f(x) is the value/image of f at x

$f : A \to B$, A is called the domain of f

The range of f is the set of all possible values of f.

x is called the ind. variable

$y = f(x)$, y is dep. variable

Ex: $f(x) = 4x^2 + 3$

plug in and make sense.

↳ What is dom.: is all real #s.

" " range: $[3, +\infty)$

↳ $4x^2 \geq 0$, $3 \geq 3 \Rightarrow f(x) \geq 3$

↳ $f(5) = 103$

$\{x \mid x \geq 0\} = [0, +\infty)$

Ex: $f(x) = \dfrac{1}{\sqrt{x}}$, Dom $= (0, +\infty) = \{x \mid x > 0\}$

Range $= (0, +\infty)$

↳ $\frac{+}{+} \Rightarrow$ only (poss.) + #s.

Defn: The net change of f from a to b is
$f(b) - f(a)$

Ex: Net change of $f(x) = |x-3|$ from -3 to 3.

$$f(3) - f(-3) = |3-3| - |-3-3|$$
$$= |0| - |-6|$$
$$= -6$$

Ex: Piece-wise

$$f(x) = \begin{cases} x^2 & , \quad x \le 0 \\ x & , \quad x > 0 \end{cases}$$

$\Rightarrow f(-1) = 1$

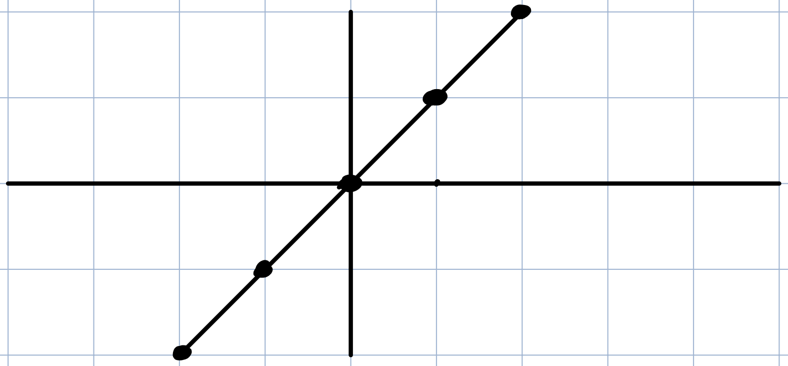
$f(2) = 2$

$f(-17) = 289 = (-17)^2$.

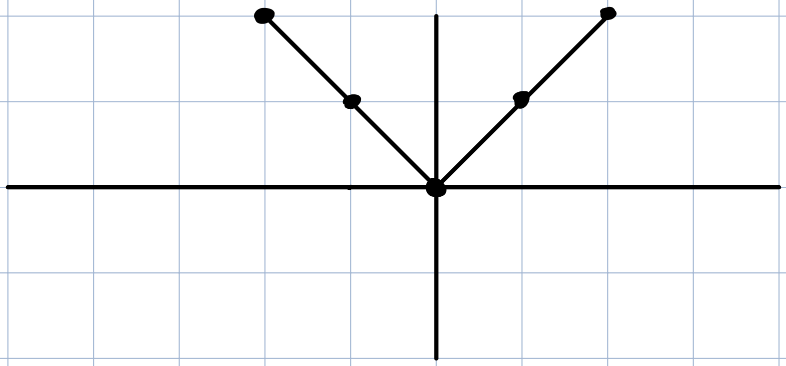
Section 1.2/1.3

Defn: The graph of $f = \{(x, f(x)) \mid x$ is in dom. of $f\}$.

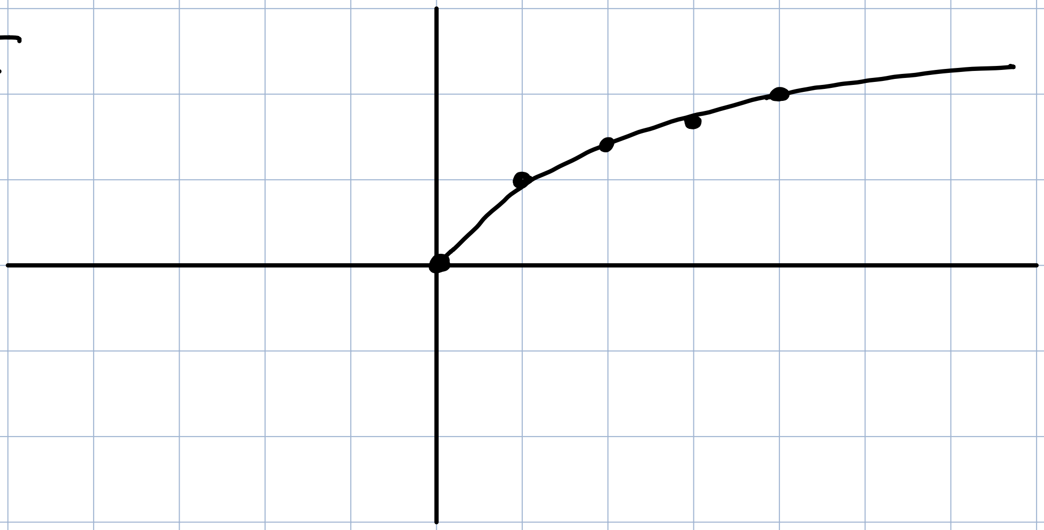
Ex: $f(x) = x$



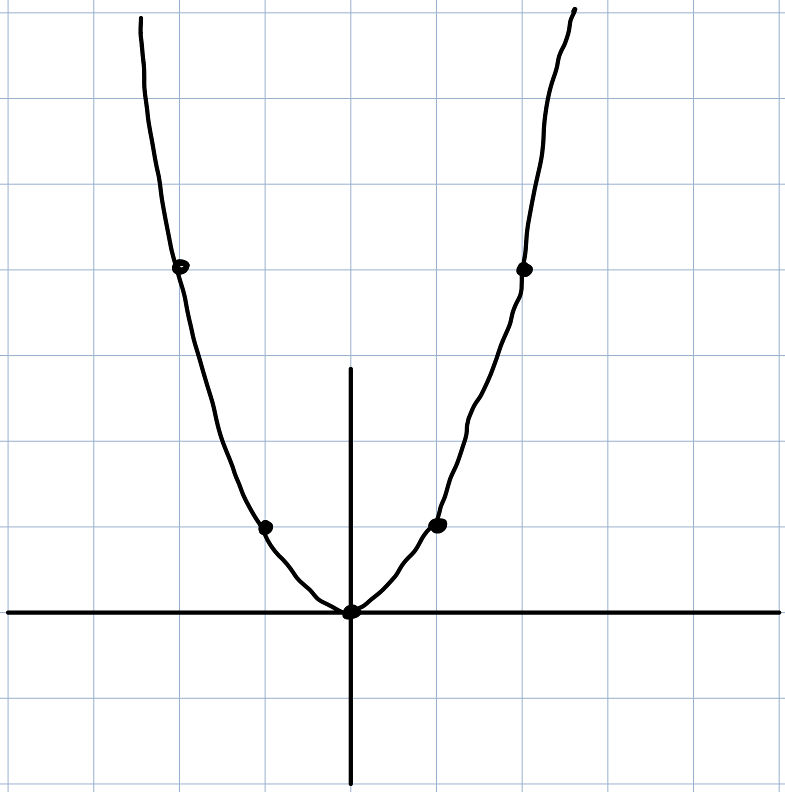
$f(x) = |x|$



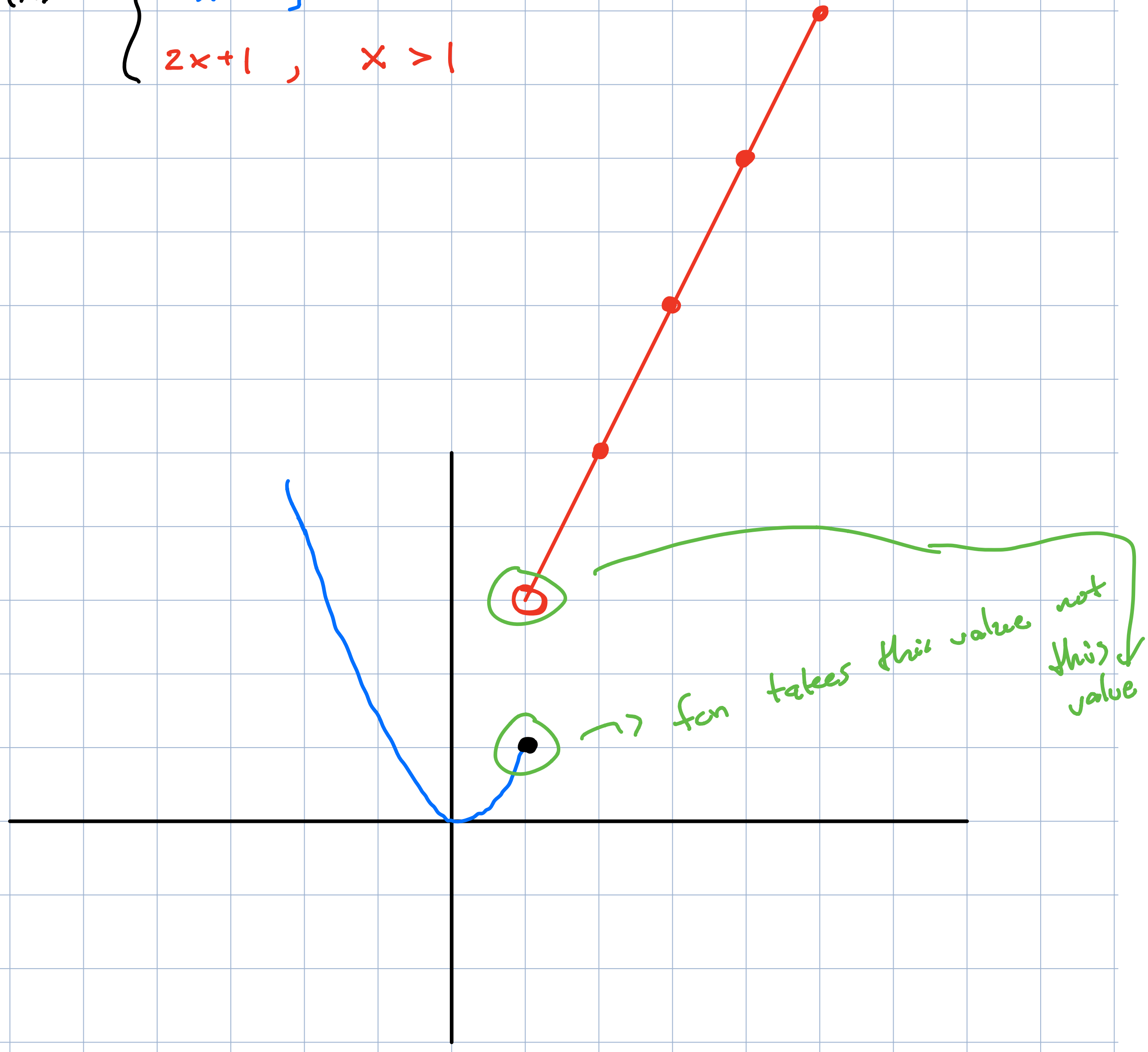
$f(x) = \sqrt{x}$



$f(x) = x^2$



Ex: $f(x) = \begin{cases} x^2 , & x \leq 1 \\ 2x+1 , & x > 1 \end{cases}$



Question: When do eqns defn fcns?

↪ $y - 2x = 0 \rightsquigarrow y = 2x , \quad f(x) = y = 2x$

↪ we could solve uniquely for y in terms of x ⇒ we get fcn

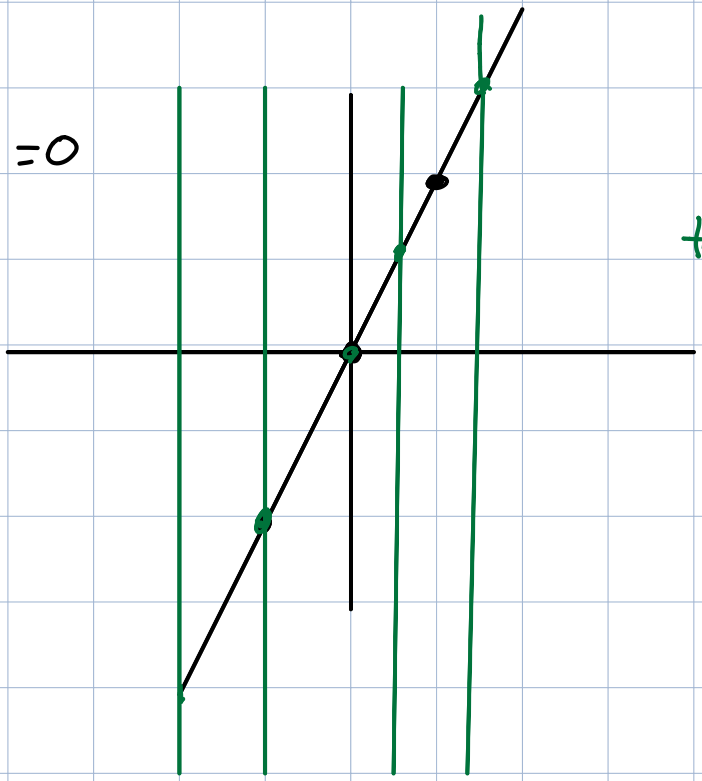
↪ $y^2 = x \rightsquigarrow y = \pm\sqrt{x}$

↪ couldn't uniquely solve ⇒ no fcn.

Rmk: Vertical line test

A graph of an eqn in x, y's defines a fcn if <u>every</u> vertical line meets the graph at most once.

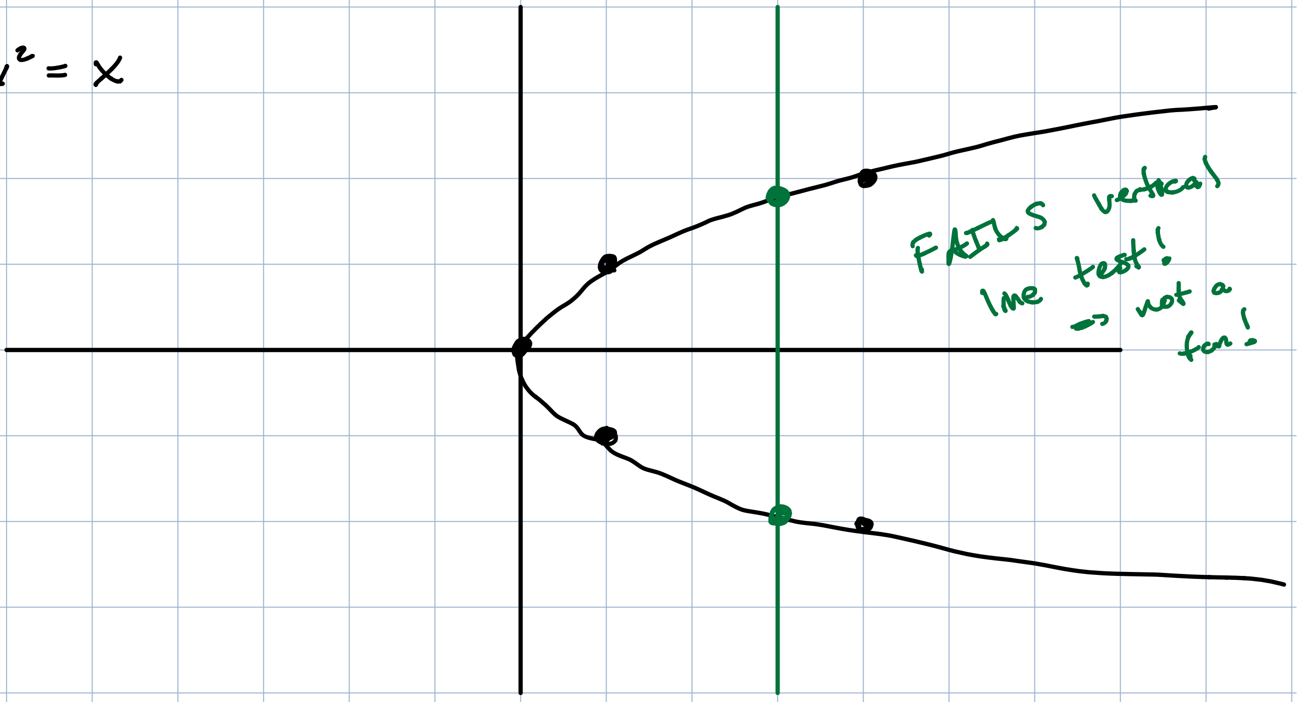
Ex: $y - 2x = 0$



Passes vert line test

$\Rightarrow$ defns a fcn.

Ex: $y^2 = x$



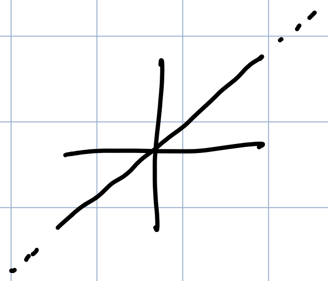
FAILS vertical line test!

$\Rightarrow$ not a fcn!

Rmk: i) height of f at x is the value of f at x.

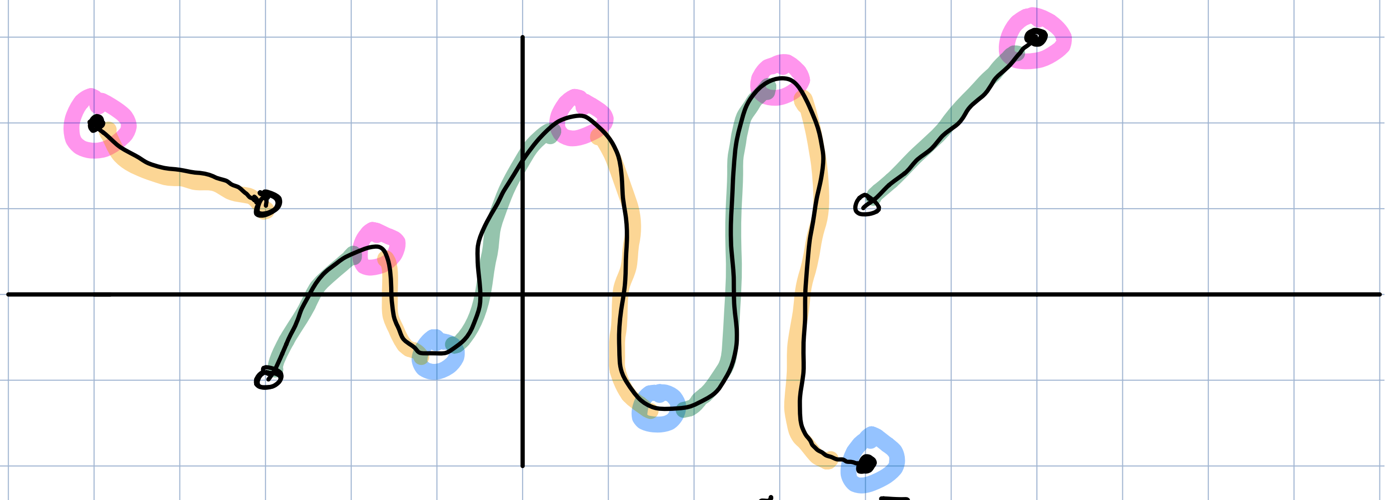
$\Rightarrow$ all possible heights = range

Ex: $f(x) = x$



range = $\mathbb{R}$

Ex:



Range = all possible heights = $[-2, 3]$

Dom. = all values x w/ the graph is above it.

$= \{x \mid -5 \le x \le 6 \text{ and } x \ne -3\}$

Defn: $f(a)$ is a loc. max. value if $f(a) \geq f(x)$ for all x "nearby" a. a is called a loc. max. 🟪

" " " " min. " " $f(a) \le f(x)$ " "

" " " " " " " " " min. 🟦

Rmk: Graphs ⇝ you tell when f is inc. versus dec.

🟪 🟩 🟧

f, g two fcn $f(x) \overset{\leq}{\geq} g(x) \Rightarrow$ graph of f at

below

x is above the graph of g at x

at the same point as