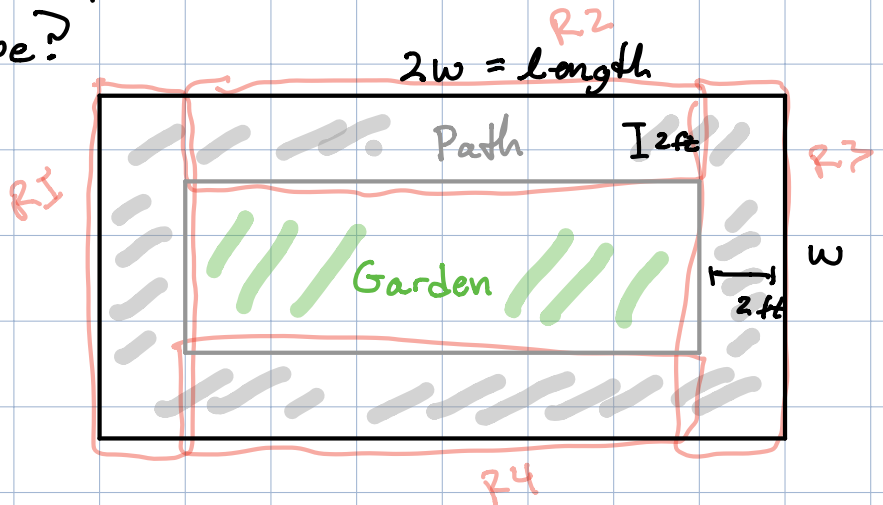
Lecture # 5

Warmups: 1) Susan wants to make a garden on a rectangular lot that's length is twice its width. She wants a path that is 2 ft. wide going around the garden. If she wants the garden to be 268 ft^2 and the lot to contain both the garden and the path, what width must the rectangular lot be?



Area of lot = $2w^2$

= area of path + area of garden

So if we solve for lengths in terms of w, we have an eqn that we can solve.

Area of yard = 268

$R1 = 2w$, $R3 = 2w$

$R2 = 2 \cdot (2w - 4) = 4w - 8 = R4$

$\Rightarrow 2w^2 = 4w + 8w - 16 + 268$

$\Leftrightarrow w^2 = 2w + 4w - 8 + 134$

$\Rightarrow w^2 = 6w + 126$

$\Rightarrow w^2 - 6w - 126 = 0$

12
126
4

Quad form $\Rightarrow$

$$w = \frac{6 \pm \sqrt{36 + 4(126)}}{2}$$

$$= 3 \pm \sqrt{540}/2$$

What is w?

Answer: $(3 + \sqrt{540}/2)$ ft.^{2}

2) If Alex can mow the lawn in 5 hours and Alex and Mike together can mow the lawn in 2 hour, then how long would it take Mike to mow the lawn by himself? $= x$

Amount mowed = (time spent mowing) $\cdot$ rate of mowing.

Alex's rate is 1 lawn / 5 hr.

Alex's rate + Mike's rate = 1 lawn / 2 hr. want to solve for x.

$$1/5 + \boxed{1/x} = 1/2$$

$1/x$ = 1 lawn / x hours = Mike's.

$$2x + 10 = 5x$$

$$\Rightarrow 3x = 10$$

$$\Rightarrow x = \frac{10}{3}$$

So Mike can mow 1 lawn in 10/3 hours.

Section 1.8: Inequalities

Rmk: Move forward like we do w/ eqn

↪ rearrange and solve; however, we need to be careful about "$-$" - signs.

↳ $x = 3 \Rightarrow -x = -3$

↳ $3 \leq 5 \not\Rightarrow -3 \leq -5$

$\Rightarrow -5 \leq -3$ or $-3 \geq -5$

↳ $x \leq 42 \Rightarrow -x \geq -42$

Ex: $4x \leq 19x - 4$

$\Rightarrow -15x \leq -4$

$\Rightarrow 15x \geq 4$

$\Rightarrow x \geq 4/15$

Ex: $4 \leq 4x - 2 < 5$

We break it up into two problems

$4 \leq 4x - 2 \Rightarrow 6 \leq 4x \Rightarrow \frac{2}{3} \leq x$

$4x - 2 < 5 \Rightarrow 4x < 7 \Rightarrow x < \frac{7}{4}$

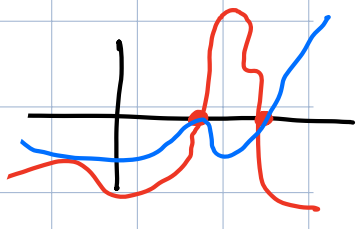
$\Rightarrow$ soln are: $\frac{2}{3} \leq x < \frac{7}{4}$

} combine results at end.

Ex: $(x-2)(x-3) \leq 0$

Step 1: Solve for when it is zero.

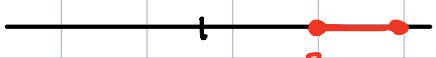
zero are $x = 2, x = 3$



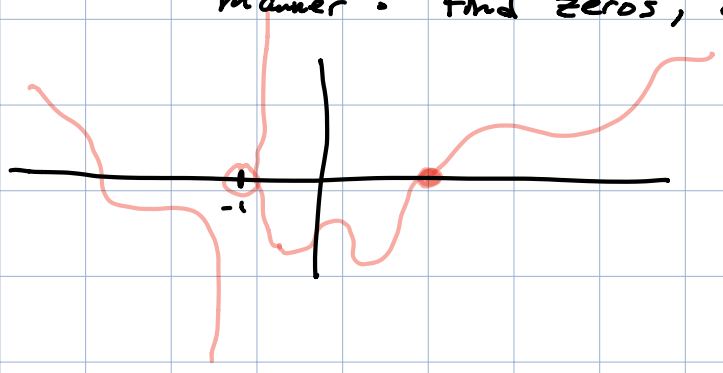
Step 2:

Region	Sign.
$x \leq 2$	+
$2 < x < 3$	−
$x \geq 3$	+

To find sign we test points in region.

Step 3: x has to be in $[2,3] =$ 

Rmk: $\frac{x-1}{x+1} > 0$ ⇝ fractional eqn, we do work in same manner: find zeros, and undefined points.



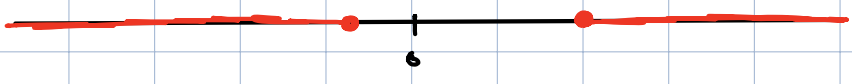
Rmk: $|x| \le c \Rightarrow -c \le x \le c$

$|x| \ge c \Rightarrow x \le -c$ or $x \ge c$

Ex: $|3x-2| \ge 4 \Rightarrow 3x-2 \le -4$ or $3x-2 \ge 4$

$\Rightarrow x \le \frac{-2}{3}$ or $x \ge 2$

$\Rightarrow$

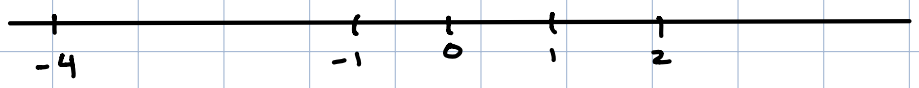


Section: The Coordinate Plane

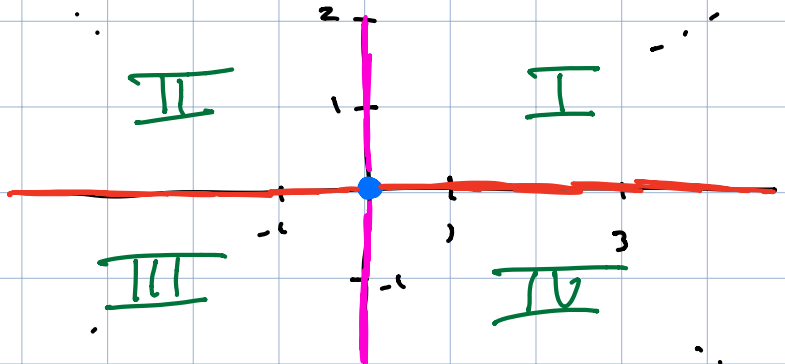
Defn: The coord. plane is the set of pairs of real numbers

$$\mathbb{R}^2 = \{(x,y) \mid x \text{ and } y \text{ are } \#s\}$$

↳ $\mathbb{R}$ =



↳ $\mathbb{R}^2$



Origin is the point $(0,0)$

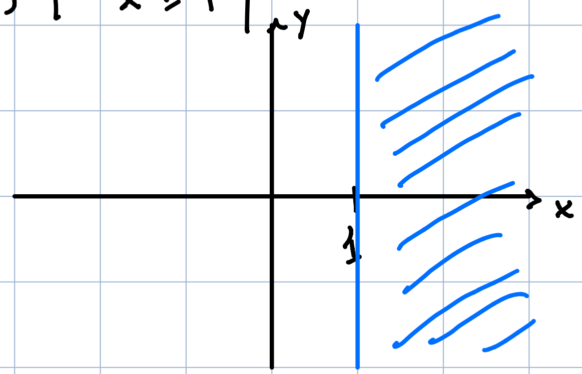
x-axis the points $(x,0)$ ranging over all x ⇝ $y=0$

y-axis " " $(0,y)$ " " " y

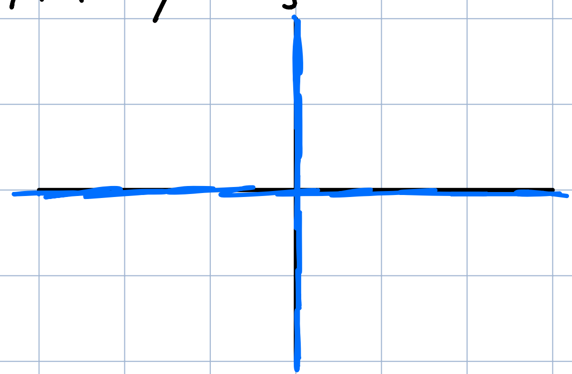
There are 4 quadrants I, II, III, IV

Ex: Drawing regions in plane

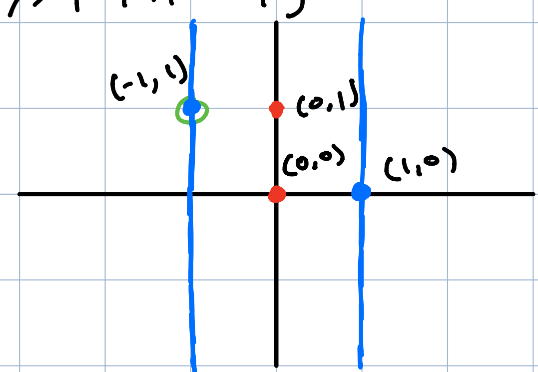
i) $\{(x,y) \mid x \geq 1\}$



ii) $\{(x,y) \mid xy = 0\}$



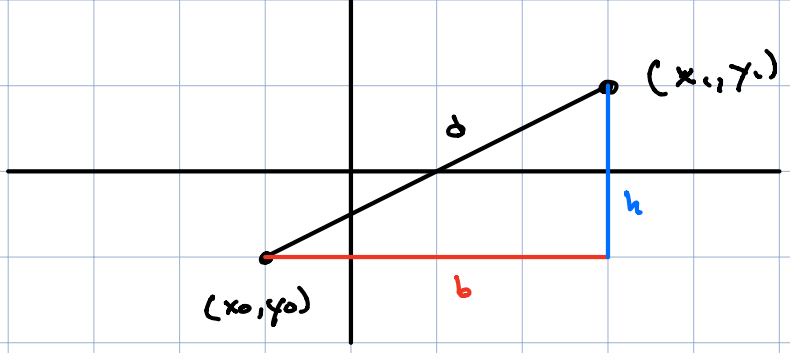
iii) $\{(x,y) \mid |x| = 1\}$



Defn: dist between two points (x_0, y_0) and (x_1, y_1) is

$$d = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$= h^2$



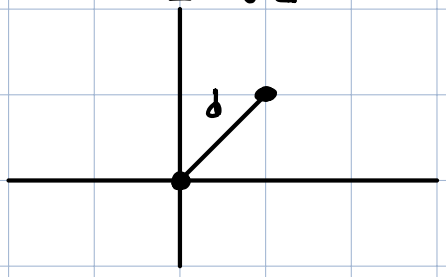
Pathag $\Rightarrow$ $d^2 = b^2 + h^2$

↳ dist of $(1,1)$ from $(0,0)$ = origin

$$\text{dist} = \sqrt{(1-0)^2 + (1-0)^2}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

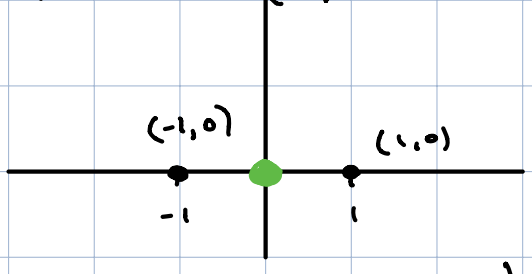


Def: Midpoint between (x_0, y_0), (x_1, y_1) is the point

$$\left(\frac{x_0 + x_1}{2}, \frac{y_0 + y_1}{2} \right)$$

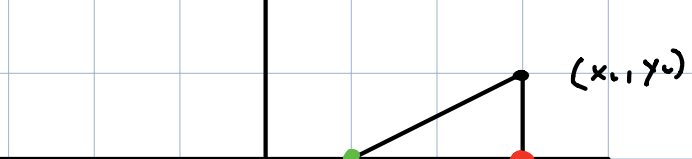
↳ average of the coordinates

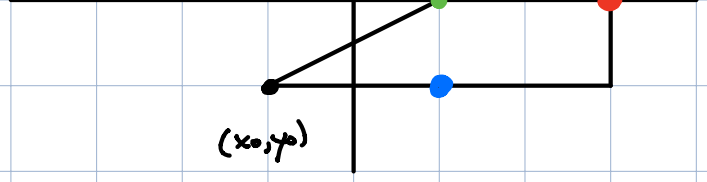
↳ Ex: $(-1,0)$ and $(1,0)$



$$MP = \left(\frac{-1+1}{2}, \frac{0+0}{2} \right) = (0,0)$$

↳

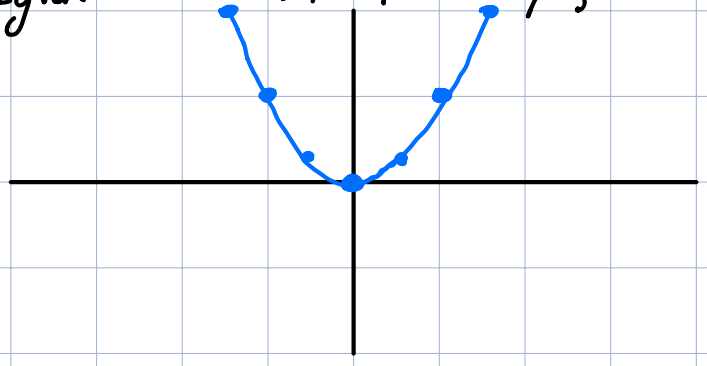




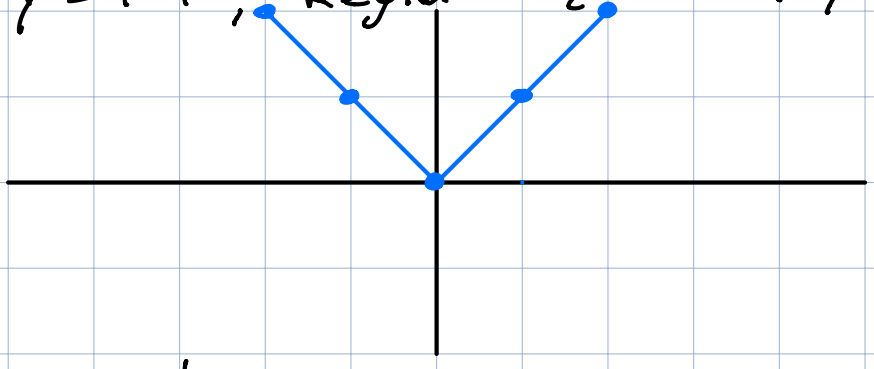
Ex: Graphs of eqns:

↳ $x^2 = y$, we can plot the pts that satisfy this eqn

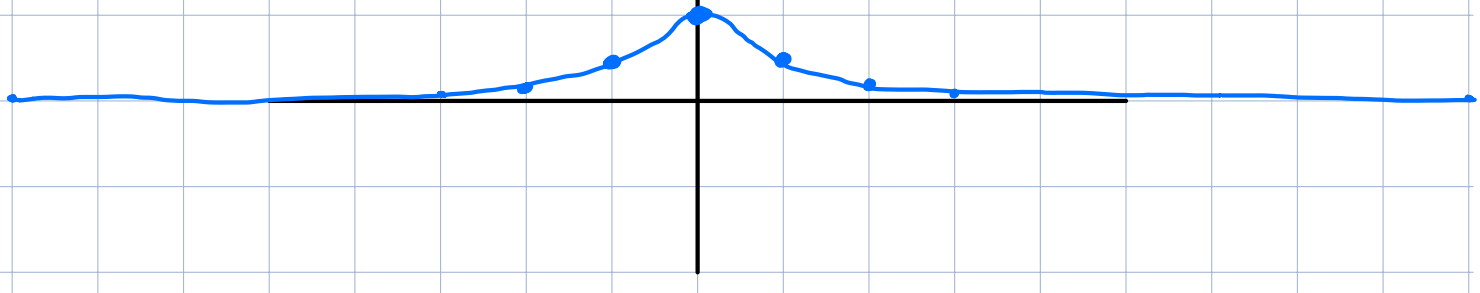
Region = $\{(x,y) \mid x^2 = y\}$



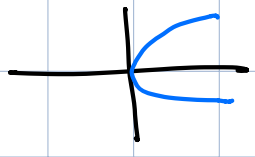
↳ $y = |x|$, Region = $\{(x,y) \mid y = |x|\}$



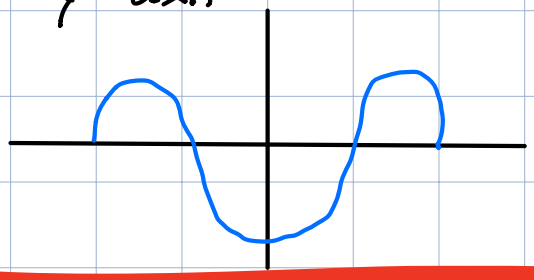
↳ $y = \frac{1}{1+x^2}$, $\{(x,y) \mid y = \frac{1}{1+x^2}\}$



Defn: Symmetric wrt x-axis if (x,y) in graph $\Rightarrow (x,-y)$ in graph



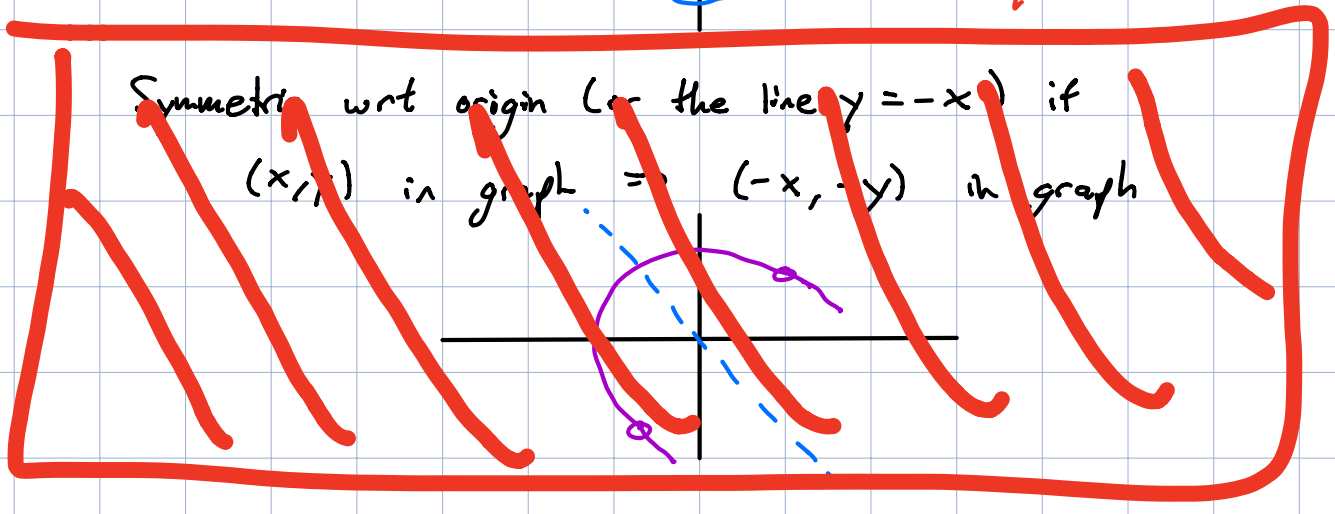
y-axis " " " " => (-x, y) " "



Incorrect. See beginning of Lecture 6 Notes!

Symmetric wrt origin (or the line $y = -x$) if

(x, y) in graph $\Rightarrow$ $(-x, -y)$ in graph



Rmk: Circle in the plane is the points that are a fixed dist r from a point (x_0, y_0)

↪ What is this region

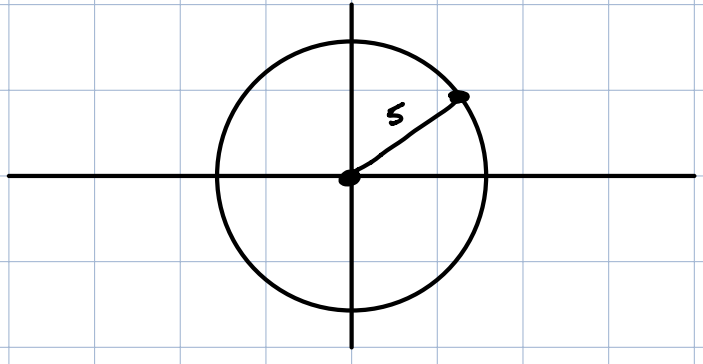
$$\{(x,y) \mid \text{dist } (x,y) \text{ to } (x_0, y_0) \text{ is } r\}$$

$$r^2 = (x - x_0)^2 + (y - y_0)^2$$

↳ Eqn of circle centered at (x_0, y_0) w/ radius r.

↳ circle of radius 5 centred at $(0, 0)$.

$$25 = x^2 + y^2$$



↪ $16 = (x-1)^2 + (y-1)^2$

$\hookrightarrow \quad 20 + x^2 - 6x + y^2 - 7x = 0$

1.1 - 1.5, 1.7 - 1.9, 2.1 - 2.3