Lecture #3

Warm-ups: 1) Simplify the expression

$$\left(\frac{2(\sqrt[3]{x})^4}{\sqrt[3]{y}}\right)^3\left(\frac{y^3}{x^{-1/2}}\right) = \left(\frac{2\,x^{4/3}}{y^{1/3}}\right)^3 y^3 x^{1/2}$$

$$= \frac{8x^4}{y^1}\cdot y^3 x^{1/2}$$

$$= 8\,x^{9/2}\,y^2$$

$x^4 \cdot x^{1/2}$

$= x^{4+\frac{1}{2}}$

$\frac{9}{2} = 4.5$

$= 4 + \frac{1}{2}$

$$\frac{1}{x^{-1/2}} = \frac{1}{\left(\frac{1}{x^{1/2}}\right)} = 1\cdot\frac{x^{1/2}}{1} = x^{1/2}$$

$$a^{-n} = \frac{1}{a^n} \iff \frac{1}{a^{-n}} = a^n$$

$$(a^n)^m = a^{n\cdot m}$$

2) Factor the expression

Trial-Error Method

$$4 + 3z - z^2 = -(z^2 - 3z - 4)$$

$$= -(z+s)(z+r)$$

$$= -(z^2 + (s+r)z + rs)$$

$$= -(z+1)(z-4)$$

$\begin{pmatrix} r = 1 \\ s = -4 \end{pmatrix}$

3) Factor the expression

factor out lowest power of z

$$4x^2y^2z^{-1/2} + 3xyz^{1/2} - z^{3/2}$$

$$= -z^{-1/2}(-4x^2y^2 - 3xyz + z^2)$$

$$= -z^{-1/2}(z+s)(z+r)$$

$$= -z^{-1/2}(z^2 + (s+r)z + rs)$$

$$= -z^{-1/2}(z + 1\cdot xy)(z - 4\cdot xy)$$

$\left(2 + \frac{-1}{2} = \frac{3}{2}\right)$

Section 1.4: Rational Expressions

Defn: A rat'l expression is a fraction / division of polynomials

↳ $\dfrac{ax+b}{cx+d}$, $\dfrac{777x^2+77x+7}{2020x^2+4040x^5}$

↳ Non-exp: $\dfrac{\sqrt{x}-x^2}{x^{57}}$

Defn: The domain of an alg. express to be the real #s that may be plugged to yield a sensible answer

↳ something/0 = undefine

↳ $\sqrt{-1}$ is not a real number

Ex:

i) $x^2-2x+1 \rightsquigarrow$ Domain $= \mathbb{R} = (-\infty, \infty)$

ii) $\dfrac{x}{(x-1)(x-5)}$; $x=1$, $\dfrac{1}{0 \cdot (-4)} = \dfrac{1}{0}$

$\Rightarrow$ 1 is not in the dom.

$x=5$, $\dfrac{5}{(5-1)(5-5} = \dfrac{5}{4 \cdot 0} = \dfrac{5}{0}$

Dom. $= \{x \mid x \neq 1, x \neq 5\}$

iii) $\dfrac{x^5}{(x-55)}$; Dom $= \{x \mid x \neq 55\}$

$55 = x$, $\dfrac{55^5}{(55-55)} = \dfrac{55^5}{0}$

$\Rightarrow$ 55 not in dom.

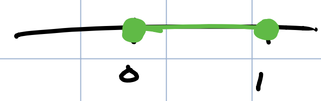
$$x = 0, \quad \frac{0}{-55} = 0$$

iv) $\sqrt{x} \cdot \sqrt{1-x}$

↳ $\sqrt{}$ of neg. #s is undefined in $\mathbb{R}$.

$\Rightarrow \quad x \geq 0$

$\Rightarrow \quad 1 - x \geq 0 \quad \Rightarrow \quad 1 \geq x$

Dom $= \{x \mid 0 \leq x \leq 1\} = [0, 1] =$ 

RmK: It makes sense to cancel terms in num/dem of rat'l exp to obtain a new rat'l express.

↳ $\frac{\cancel{2} \cdot 3}{\cancel{2} \cdot 7} = \frac{3}{7}$; $\frac{6}{14} = \frac{\cancel{2} \cdot 3}{\cancel{2} \cdot 7}$ $\left(\frac{1}{x^2 - 7x + 10} \right)$

↳ $\frac{\cancel{(x-5)}(x-1)}{(x-2)\cancel{(x-5)}} = \frac{(x-1)}{(x-2)}$

↳ $\frac{x^2 - 1}{x^2 - 2x + 1} = \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}(x-1)} = \frac{x+1}{x-1}$

RmK: We can mult/divide rat'l expression

↳ $\frac{x-1}{x-2} \cdot \frac{x-3}{x-4} = \frac{(x-1)(x-3)}{(x-2)(x-4)}$

↳ $\frac{x-1}{x-2} \div \frac{x-1}{x-2} = \frac{\cancel{x-1}}{\cancel{x-2}} \cdot \frac{\cancel{x-2}}{\cancel{x-1}} = 1$

↳ $\frac{2}{3} \div \frac{3}{4} = \frac{2/3}{3/4} = \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$

RmK: Rat'l denominator that have radials

A, B, C = alg expression

↳ $A = x^2 - 1$, $B = x^2$, $C = \pi - 77x^5$

$$\frac{A}{\sqrt{B} + \sqrt{C}} \cdot \frac{\sqrt{B} - \sqrt{C}}{\sqrt{B} - \sqrt{C}} = \frac{A\sqrt{B} - A\sqrt{C}}{B - C}$$

Remove $\sqrt{\ }$

↳ $$\frac{x^2 - 1}{\sqrt{x^2} + \sqrt{\pi - 77x^5}}$$

$$(\sqrt{B} + \sqrt{C})(\sqrt{B} - \sqrt{C})$$

$$= B - \sqrt{B}\sqrt{C} + \sqrt{B}\sqrt{C} - C$$

$$= B - C$$

$2 + \frac{1}{2}$

$= \frac{4}{2} + \frac{1}{2}$

$= 5/2$

$x^{1/2} \cdot z^{1/2} = \sqrt{x} \cdot \sqrt{z} =$

↳ $$\frac{x^2}{\sqrt{y} + \sqrt{xz}} \cdot \frac{\sqrt{y} - \sqrt{xz}}{\sqrt{y} - \sqrt{xz}} = \frac{x^2 y^{1/2} - x^{5/2} z^{1/2}}{y - xz}$$

Section 1.5: Equations

Defn: Eqn is a math statement of equality.
When variable/letters, the values of the var. that make the eqn hold are called roots (solutions)

Ex: i) $3 + 5 = 8$

$x = 1$, $3 \cdot 1 + 5 = 8$

ii) $3x + 5 = 8$ (x in $\mathbb{R}$ st this eqn holds)

iii) $$\frac{\sqrt{w} - z^2 x^{77}}{\sqrt[3]{z^{33}}} = w$$

$w = 1$, $x = 0$, $z = 1$

iv) $(x-1)(x-1) = x^2 - 2x + 1 = 0$

Rule: A, B, C = alg exp.

$A = x$
$B = x^2$
$C = y$

↳ $\underset{-A}{A} + B = \underset{-A}{A} + C \iff B = C$

↳ $\underset{-x}{x} + x^2 = \underset{-x}{x} + y \iff x^2 = y$

$\underset{-3}{3} + \pi = \underset{-3}{3} + \pi \iff \pi = \pi$

↳ $A = B \iff A \cdot C = B \cdot C \quad (C \neq 0)$.

↳ $x = 7$, $C = 42 \iff 42x = 42 \cdot 7$

↳ $AB = 0 \implies A = 0$ or/and $B = 0$

↳ $(x-2)(x-1) = 0 \implies x - 2 = 0$ or $x - 1 = 0$

$x = 2$

Ex: Solve Eqn (find values of the letter st we get =)

↳ $3x - 2 = 0 \iff 3x = 2 \iff x = 2/3$

So $\frac{2}{3}$ is the solution

↳ $x^2 - 5x + 6 = 0$, what are the roots?

Find factors?

$(x-2)(x-3) = 0$

$\Rightarrow$ roots are $2, 3$

↳ quad poly $= 0$ is called quad eqn

Rmk: Completing the square

$\hookrightarrow$ $x^2 + 3x - 16 = 0$

$\Rightarrow x^2 + 3x = 16$

Want is a = real # st

$\underline{x^2 + 3x + a = (x+b)^2 = (x^2 + 2bx + b^2)}$

for some real # b

If true, then

$\Rightarrow (x+b)^2 = x^2 + 3x + a = 16 + a$

$\Rightarrow x + b = \pm\sqrt{16+a}$

$\Rightarrow x = \pm\sqrt{16+a} - b$

$2b = 3$, $a = b^2$

$\Rightarrow b = \frac{3}{2}$, $a = 9/2$,

So plug in, $x = \pm\sqrt{16 + \frac{9}{2}} - \frac{3}{2}$

$(x+b)^2 = 16$

$(x+b) = \pm\sqrt{16}$

$x = \pm\sqrt{16} - b$

$$x^2 + 3x + \frac{9}{2} = 16 + \frac{9}{2}$$

$$\overset{\shortparallel}{\left(x+\frac{3}{2}\right)\left(x+\frac{3}{2}\right)}$$

$$\shortparallel$$

$$(x + 3/2)^2$$

$\sqrt{\ }$ both sides, $x + \frac{3}{2} = \pm\sqrt{16 + 9/2}$