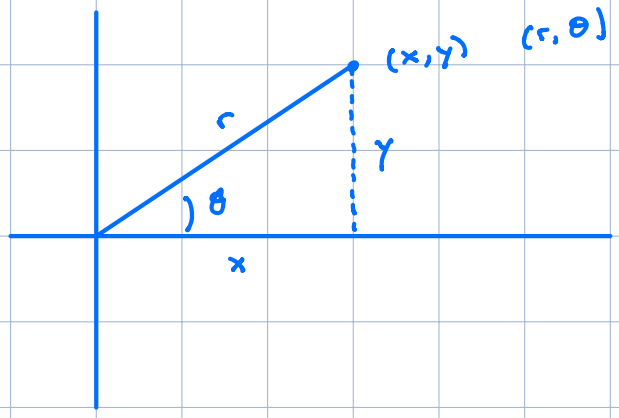
Lecture # 24

Warm-up: 1) Convert the following equation from polar coords to Cart. coords:

$$r = \frac{\tan(\theta)}{\cos(\theta) + 1}$$

↪



$r\sin(\theta) = y$

$r\cos(\theta) = x$

$\tan(\theta) = y/x$

$r^2 = x^2 + y^2$.

$$r = \frac{\tan(\theta)}{\cos(\theta) + 1}$$

$\Rightarrow \quad r\cos(\theta) + r = \tan(\theta)$

$\Rightarrow \quad x + r = y/x$

$\Rightarrow \quad r = \frac{y}{x} - x$

$\Rightarrow \quad r^2 = (y/x - x)^2$

$\Rightarrow \quad x^2 + y^2 = (y/x - x)^2$.

Topic: Graphing Fcns in General

1) x-int, y-int.

2) Asymptotes (ver, hor.)

↪ tan has ver asym when cos = 0

cot " " " " sin = 0

↪ Rat'l fcn ⇝ ver asy ⟷ zeros of the denom.

↳ $\log_a(x)$ has a ver asy along $x = 0$.

↳ Rat'l fcn have hor. asym.

↳ a^x has a hor asym along $y = 0$ as $x \to -\infty$ $(a > 1)$.

3) Sign (is graph above or below x-axis)

↳ test points.

4) End behavior

↳ what happens as $x \to +\infty$

" " " $x \to -\infty$.

↳ odd deg poly w/ lead coef 1

as $x \to +\infty$, $f(x) \to +\infty$

$x \to -\infty$, $f(x) \to -\infty$.

↳ $x^5 + 22x - 7$

$a \cdot x^n$ (sign of a will poss flip graph)

as $x \to +\infty$, $x^2 \to +\infty$

" " " , $-2x^2 \to -\infty$.

as $x \to -\infty$, $x^3 \to -\infty$.

" " " , $-2x^3 \to +\infty$.

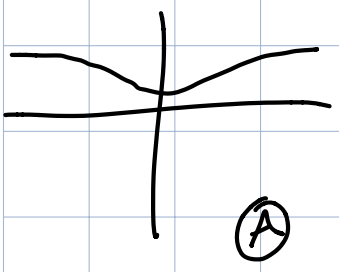
↳ a^x, as $x \to +\infty$, $a^x \to +\infty$

$\log_a(x)$ as $x \to +\infty$, $\log_a(x) \to +\infty$.

5) Domain and range.

Ex: $\log(x^2 + 2x - 2) = f$, what does the graph look like.

↳ End behavior: as $x \to -\infty$.



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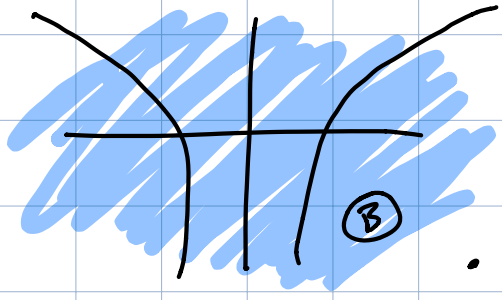


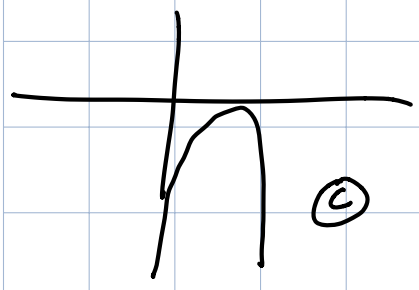
Ⓑ

$x^2 \to +\infty$

$\Rightarrow x^2+2x-2 \to +\infty.$

$\to \log(x^2+2x-2) \to +\infty.$

• as $x \to +\infty.$



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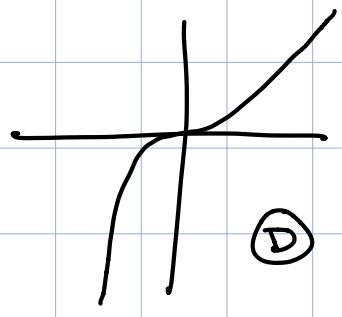


Ⓓ

$x^2+2x-2 \to +\infty.$

$\Rightarrow \log(x^2+2x-2) \to +\infty.$

↪ Dom: $\log$(numbers < 0) is undefined.

So is $x^2+2x-2 < 0$ for some x.

For $x=0$, $\left(\frac{1}{2}\right) < 0$

$\Rightarrow f(0)$ is undefined.

Topic: Graphing Rat'l Functions

Important feature: intercepts, dom, range?, end behav. asymptotes, sign

Ex: $\dfrac{(2x+2)(x+2)}{(x-1)(x-2)} = f(x)$

• x-int = $x=-2$ or $x=-1$

• y-int = plug in 0! $(0, f(0))$

$f(0) = \dfrac{2\cdot 2}{(-1)\cdot(-2)} = 2.$

$\Rightarrow (0,2)$

• ver asym = $x=1$ or $x=2$.

• hor asym = $y=2$

↳ $P/Q = f$

If $\deg(P) < \deg(Q) \Rightarrow$ $y = 0$ is hor. asym.

If $\deg(P) = \deg(Q) \Rightarrow$ $y = \frac{\text{leading coeff } P}{\text{leading coeff } Q}$ is hor asym.

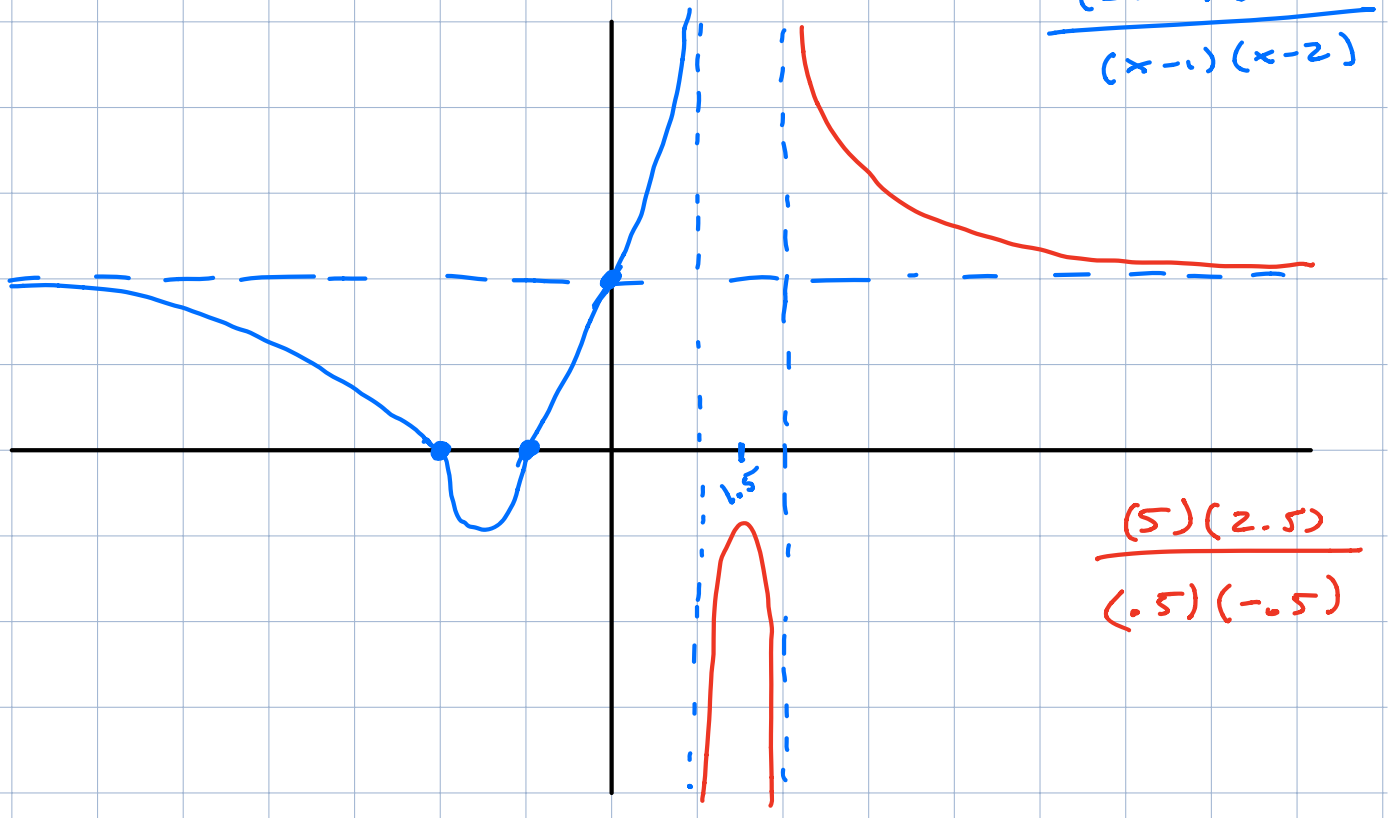
If $\deg(P) > \deg(Q) \Rightarrow$ no hor asym.

end beh. of f is end beh. of P up to a sign

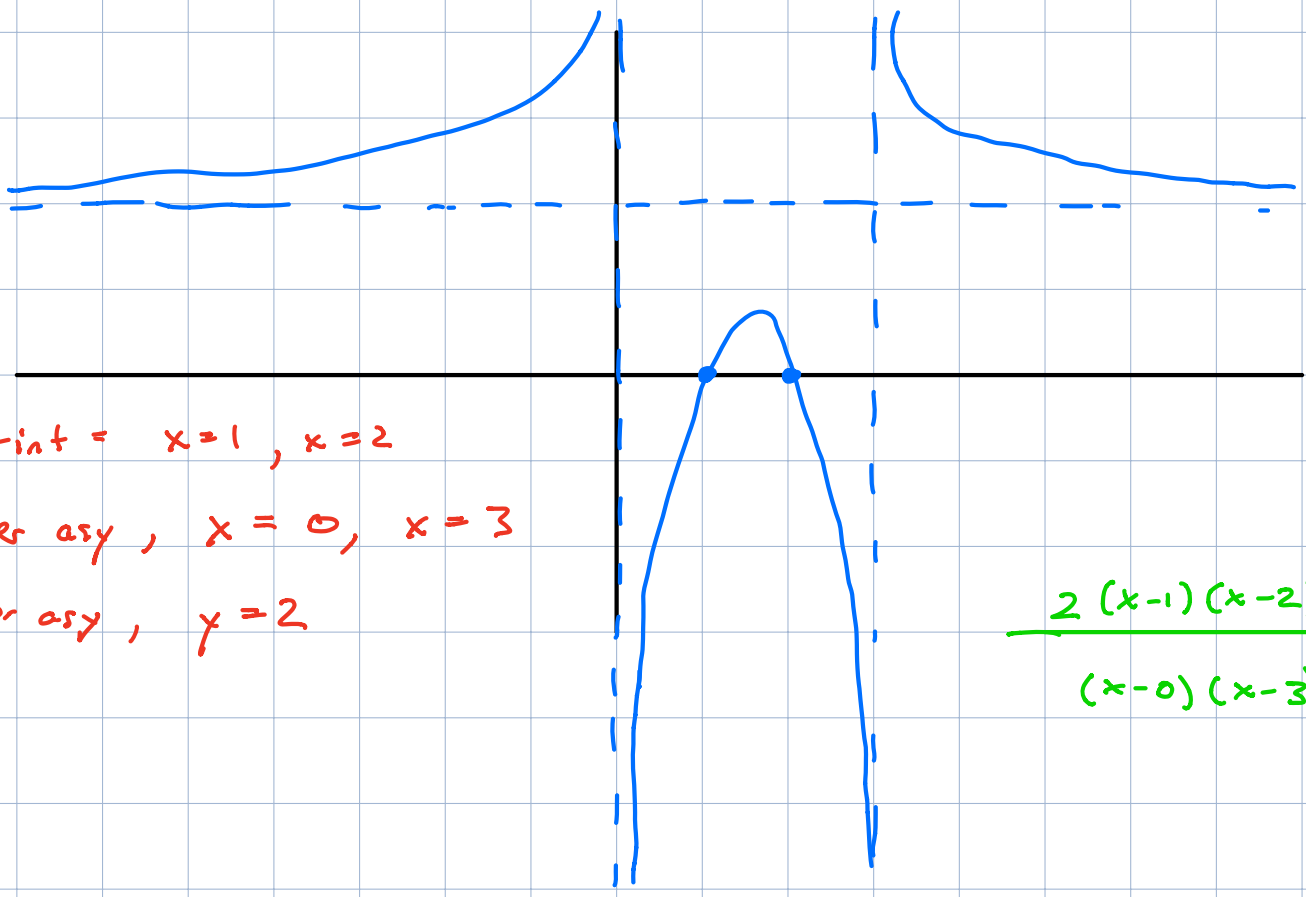
↳ $x \to +\infty$, $\frac{x^3}{(x-1)} \to +\infty$

$x \to -\infty$ $\frac{x^3}{(1-x)} \to -\infty$.

- $\text{dom}(f) = \{x \mid x \neq 1, 2\}$.



Ex: Given the graph of the fcn f



x-int = $x=1$, $x=2$

Ver asy, $x=0$, $x=3$

hor asy, $y=2$

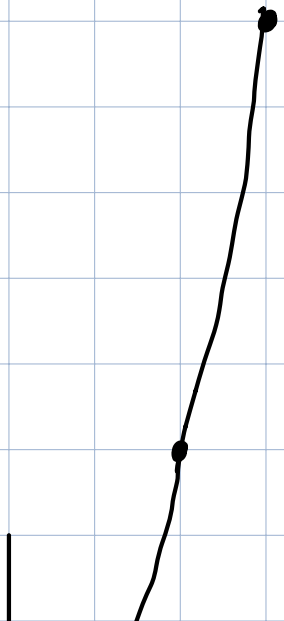
$$\frac{2(x-1)(x-2)}{(x-0)(x-3)}$$

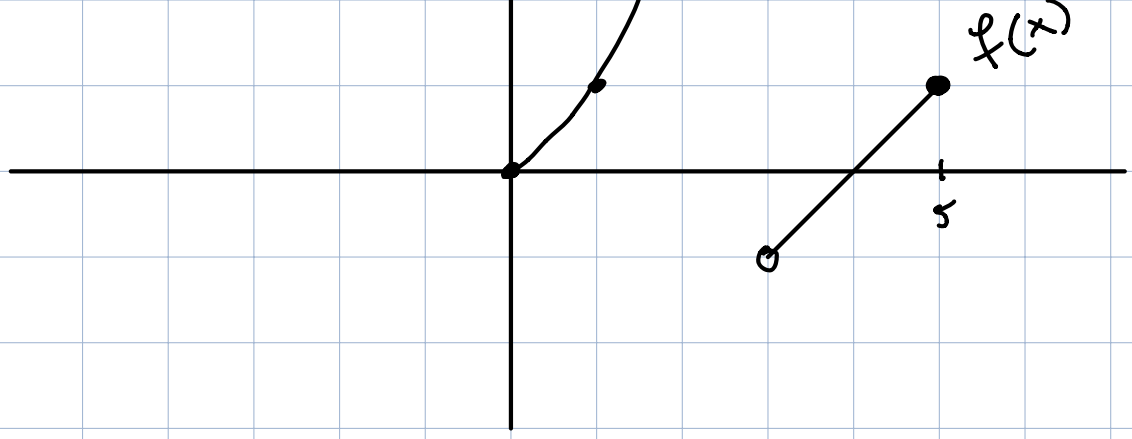
Give a rat'l fcn w/ the same intercepts, asym and signs of the fcn f.

Ex:

$$f(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ x-4, & 3 < x \le 5 \end{cases}$$

Midterm 1
3.a, 7.10.

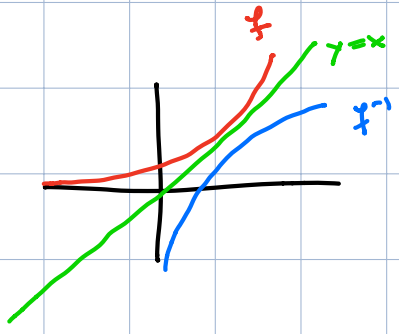




Topic: Inverses

1) Fcn is 1-to-1 iff it passes hor. line test. iff Fcn is invertible.

2) Graph f^{-1} is the graph of f reflected across $y = x$.



3) Ex: $$f(x) = \frac{1+x}{2+3x}$$

Solve for inverse.

① Replace $y = f(x)$

$$y = \frac{1+x}{2+3x}$$

② Solve for x in terms of y.

$$\Rightarrow (2+3x)(y) = 1+x$$

$$\Rightarrow 2y + 3xy = 1+x$$

$$\Rightarrow 2y - 1 = x - 3xy = x(1-3y)$$

$$\Rightarrow x = \frac{2y-1}{1-3y}$$

③ Set $f^{-1}(x) = (\quad)$.

$$f^{-1}(y) = \frac{2y-1}{1-3y}.$$

Ex: $$f(x) = \frac{1+x^3}{2+3x^3}$$

$$\Rightarrow x^3 = \frac{2y-1}{1-3y}$$

$$\Rightarrow x = \sqrt[3]{\frac{2y-1}{1-3y}}.$$

Ex: Find inverse of

$$f(x) = \log_e\left(\frac{1+x}{2+3x}\right).$$

Topic: 1) $\log_a(x)$ is the inverse of a^x.

↳ i) $a^{\log_a(x)} = x$

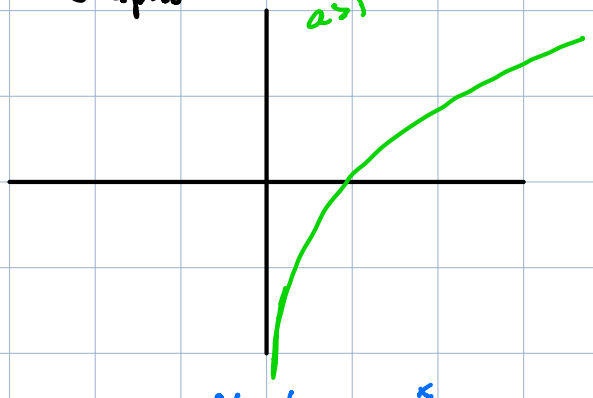
ii) $\log_a(a^x) = x$

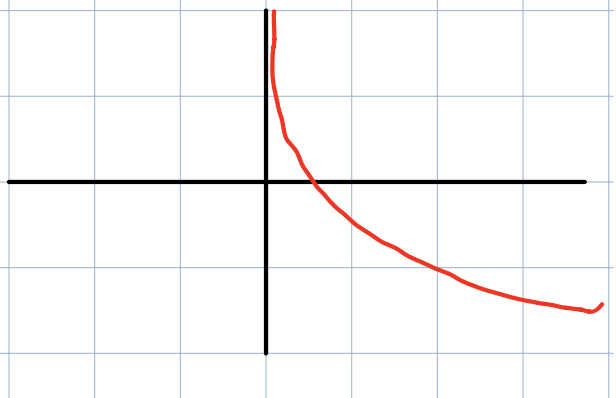
iii) $\log_a(1) = 0$. , $a^0 = 1$.

$(a<1$ $(1/2)^x = \frac{1}{2^x}$

$1>a>0$

2) Graphs





↳ reflect. a^x

3) $\log_a(A \cdot B) = \log_a(A) + \log_a(B)$

$\log_a(A/B) = \log_a(A) - \log_a(B)$

$\log_a(A^c) = C \cdot \log_a(A)$

Warning: $\log(A+B) \neq \log(A) + \log(B)$.

↳ Combine everything to a single expression

$$2\log(a) - \log(b) + \log(a^2).$$

$$= \log(a^2) - \log(b) + \log(a^2)$$

$$= \log(a^2 \cdot a^2) - \log(b)$$

$$= \log(a^4) - \log(b)$$

$$= \log(a^4/b).$$