Lecture # 23

$\to$ Midterm 2
Exer 3.4

Warm-up: Put the quadratic fcn

$$f(x) = -2x^2 + \frac{4}{3}x + \frac{77}{18}$$

in std form. Use this form to compute the vertex, the x-int., and the range.

$$-2\cdot\left(\frac{-77}{36}\right) = \frac{-2\cdot -77}{18\cdot 2}$$

$\hookrightarrow$ std $f(x) = a\cdot(x-h)^2 + k$

$$f(x) = -2\left(x^2 - \frac{2}{3}x - \frac{77}{36}\right)$$

$(x+b)^2$.

$$= -2\left(\underline{x^2 - \frac{2}{3}x + a} - a - \frac{77}{36}\right)$$

$$(x+b)(x+b) = \left(x-\frac{1}{3}\right)\left(x-\frac{1}{3}\right)$$

$$= x^2 - \frac{2}{3}x + \frac{1}{9}$$

$a = \frac{1}{9}$

$$= -2\left(x^2 - \frac{2}{3}x + \frac{1}{9} - \frac{1}{9} - \frac{77}{36}\right)$$

$$= -2\left(\left(x-\frac{1}{3}\right)^2 - 81/36\right)$$

$$\frac{-4}{36} - \frac{77}{36}$$

$$= -2\cdot\left(x-\frac{1}{3}\right)^2 + 81/18$$

$=$ std form of f.

① Vertex $= (h,k) = (1/3, 81/18)$

② $0 = -2\,(x - 1/3)^2 + 81/18$

$\Rightarrow 0 = (x-1/3)^2 - 81/36$

$\Rightarrow (x-1/3)^2 = 81/36$

$\Rightarrow x = \pm\sqrt{81/36} + 1/3$

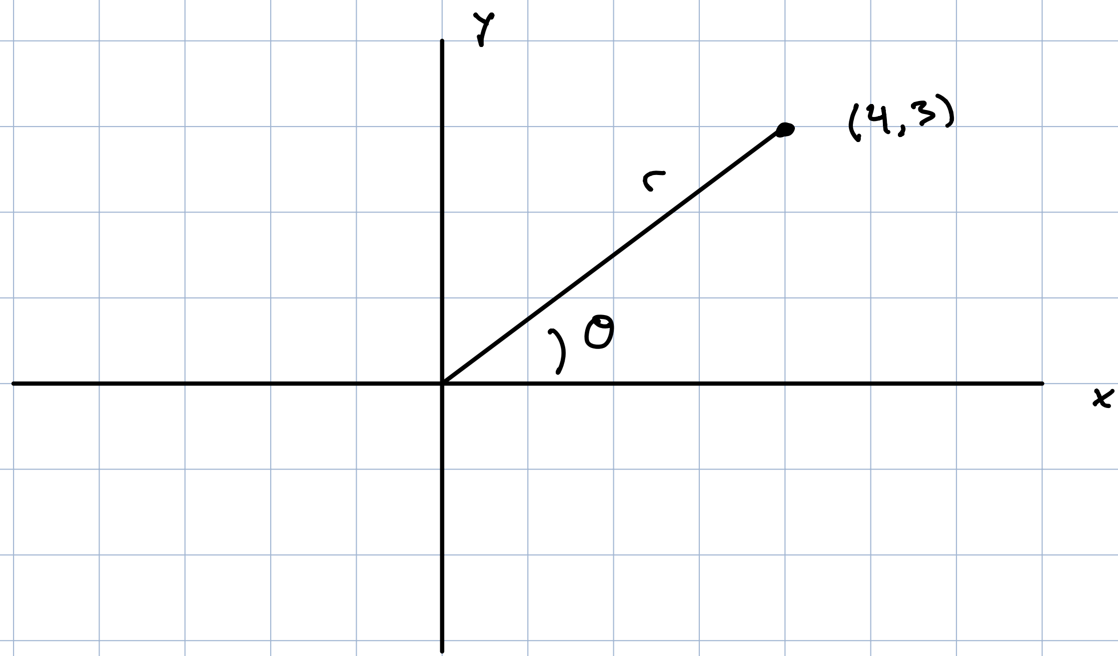
$= \pm 9/6 + 1/3$.

③ If $a < 0 \Rightarrow (-\infty, k]$

" $a > 0 \Rightarrow [k, +\infty)$

Section 8.1: Polar Coordinates.

Motivation:



Defn: Polar coords are pairs (r, θ) where θ is an angle wrt the x-axis and r is a "distance" from the origin.

Ex:

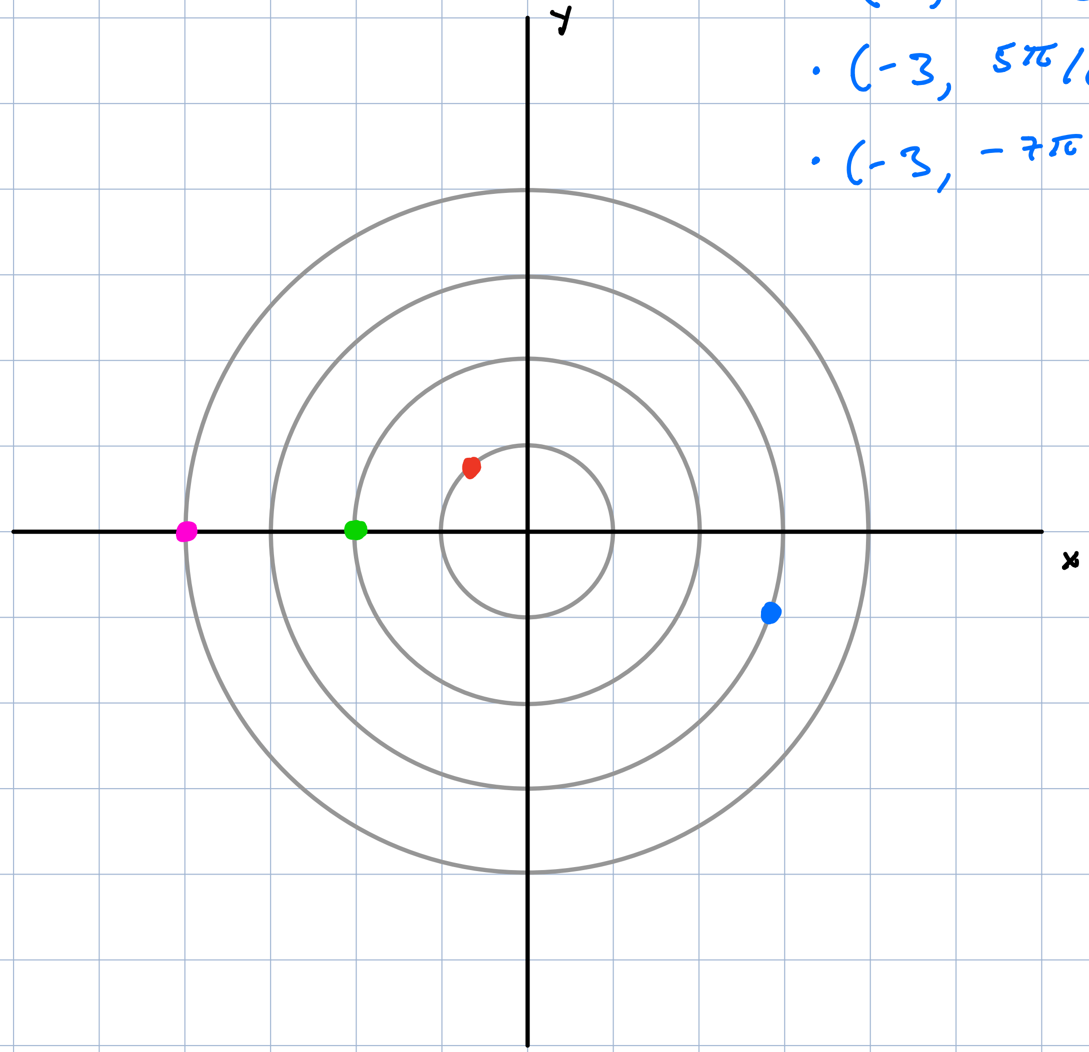
① $(1, 3\pi/4) = (-1, -\pi/4)$

② $(3, -\pi/6) =$ 3 diff polar coords that rep the point.

- $(3, -\pi/6 + 2\pi)$
- $(3, 11\pi/6)$
- $(-3, 5\pi/6)$
- $(-3, -7\pi/6)$

③ $(2, 3\pi) = (2, \pi)$

④ $(-4, 2\pi) = (4, \pi)$



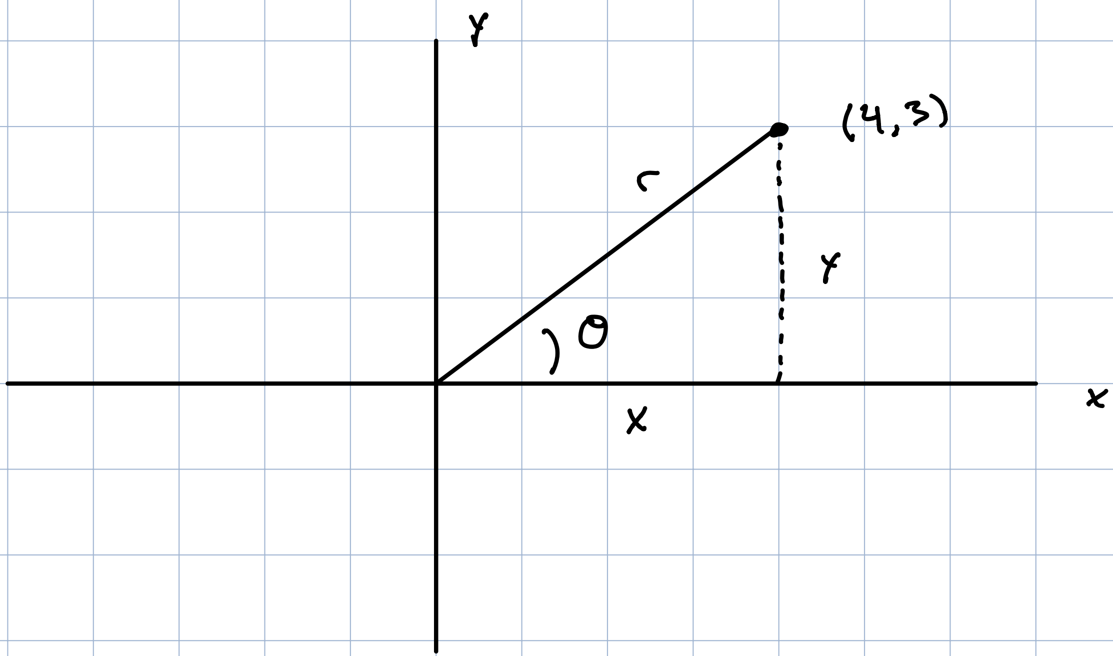
Rem: ① Cartesian Coords ⇝ gives left/right and up/down

" x-coord   " y-coord

② Polar Coords ⇝ gives direction and "distance"

" θ-coord   " r-coord.

Ex: Convert from Polar to Cart.



$\sin(\theta) = 3/r$, $\cos(\theta) = 4/r$

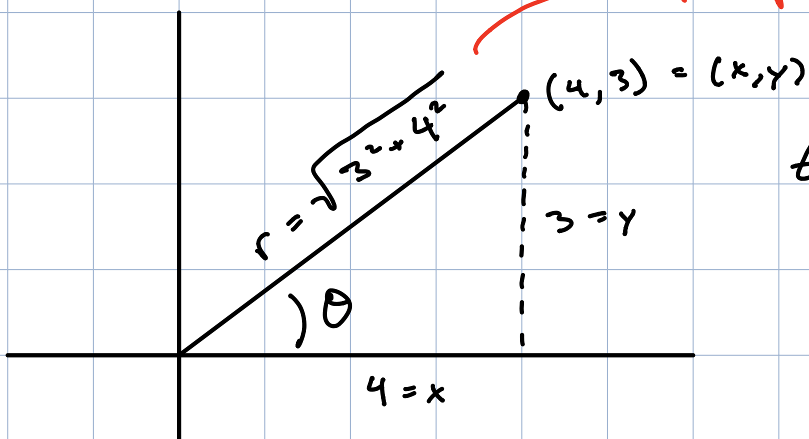
$\Rightarrow$ $3 = r\sin(\theta)$, $4 = r\cos(\theta)$.

Fact: $x = r\cos(\theta)$

$y = r\sin(\theta)$.

Ex: Cart to Polar

$r^2 = 3^2 + 4^2$.



$\tan(\theta) = 3/4$

Fact: $r^2 = x^2 + y^2$

$\tan(\theta) = y/x$

$x = r\cos(\theta)$

$y = r\sin(\theta)$

Ex: Convert into Cart coords:

$(1, 3\pi/4) \rightsquigarrow (1 \cdot \cos(3\pi/4), 1 \cdot \sin(3\pi/4))$

$= (-\sqrt{2}/2, \sqrt{2}/2).$

$(3, -\pi/6) \rightsquigarrow (3 \cdot \cos(-\pi/6), 3 \cdot \sin(-\pi/6))$

$= (3\sqrt{3}/2, -3/2).$

$(2, 3\pi) \rightsquigarrow (2 \cdot (-1), 2 \cdot (0)) = (-2, 0).$

$(-4, 2\pi) \rightsquigarrow (-4 \cdot \cos(2\pi), -4 \cdot \sin(2\pi)).$

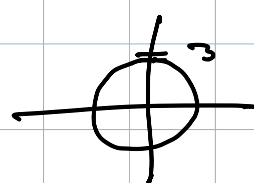
$= (-4, 0).$

Rem: We can express eqn in x, y (ie in Cart) as eqn in (r, θ) (ie polar).

Ex: ① $x^2 + y^2 = 9 \rightsquigarrow r^2 = 9 \Rightarrow r = \pm 3.$

$\{(x,y) \mid x^2 + y^2 = 9\}$ " circle of radius 3

$\{(r,\theta) \mid r = \pm 3\}.$ " circle of radius 3.



② $x^2 = y \rightsquigarrow x = r\cos(\theta), \ y = r\sin(\theta)$

$\Rightarrow r^2\cos^2(\theta) = r\sin(\theta)$

$\Rightarrow r = \tan(\theta)/\cos(\theta).$

Ex: Polar to Cartesian

$x = r\cos\theta$

$y = r\sin\theta$

$\tan(\theta) = y/x$

$r^2 = x^2+y^2$

① $r = 5\sec(\theta)$

$\Rightarrow r = 5/\cos(\theta)$

$\Rightarrow 5 = r\cos(\theta) = x$

$\Rightarrow 5 = x.$

② $r = 7\cdot\sin(\theta)$

$\Rightarrow r^2 = 7\cdot r\sin(\theta)$ mult both sides by r

$\Rightarrow x^2+y^2 = 7y$

$\Rightarrow x^2+y^2-7y = 0$

$\Rightarrow x^2 + y^2 - 7y + \frac{49}{4} = \frac{49}{4}$

$\Rightarrow x^2 + (y-7/2)^2 = 49/4$

$=$ circle of radius $\sqrt{49/4} = 7/2$

centered at $(0, 7/2)$.

Ex: Convert the eqn from polar to cartesian:

$r = 5\csc(\theta) = 5/\sin(\theta)$

$\Rightarrow r\sin(\theta) = 5$

$\Rightarrow y = 5$

Convert the eqn from cart to polar:

$x^2+y^2-2y+x = 0.$

$x = r\cos\theta$

$y = r\sin\theta$

$\tan(\theta) = y/x$

$r^2 = x^2+y^2$

$\Rightarrow r^2 - 2y + x = 0$

$\Rightarrow r^2 - 2y + r\cos(\theta) = 0$

$\Rightarrow r^2 - 2r\sin(\theta) + r\cos(\theta) = 0$

$$(r\cos(\theta))^2 + (r\sin(\theta))^2$$
$$= r^2\cos^2\theta + r^2\sin^2\theta$$
$$= r^2(\cos^2\theta + \sin^2\theta)$$
$$= r^2.$$

Review: f, g be two fcns.

$\hookrightarrow (f \circ g)$, $(f \circ g)(x) = f(g(x))$ $\rightsquigarrow$ Composition

$(f \cdot g)$, $(f \cdot g)(x) = f(x) \cdot g(x)$. $\rightsquigarrow$ Multiplicatio