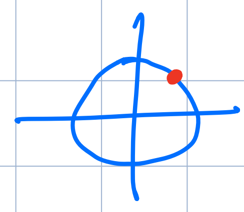
Lecture # 22

Warm-Up: 1) Find all real solutions to the equations:

a) $\sqrt{2}\,\sin(x) - 1 = 0$

b) $\sin^2(x) + 6\sin(x) - 7 = 0$



a) $\sin(x) = 1/\sqrt{2} = \sqrt{2}/2$

Strategy: ① Solve for all soln on $[0, 2\pi)$

$x = \pi/4$, $x = 3\pi/4$

② Add multi of 2π to obtain all solns.

$\Rightarrow$ $x = \pi/4 + 2\pi n$

$x = 3\pi/4 + 2\pi n.$

b) $\sin^2(x) + 6\sin(x) - 7 = 0 \xrightarrow[2]{\sin(x) = y} y^2 + 6y - 7 = 0$

$(\sin(x) + 7)(\sin(x) - 1) = 0 \qquad (y + 7)(y - 1) = 0$

Solve ① $\sin(x) + 7 = 0$ and ② $\sin(x) - 1 = 0.$

① No solns, why? $\text{range}(\sin(x)) = [-1, 1].$

② $\sin(x) = 1$

Solve for all soln on $[0, 2\pi)$

$x = \pi/2$

Add multi of 2π to obtain all soln

$x = \pi/2 + 2\pi \cdot n.$ n = integer.

2) If $\cot(x) = 1/2$ and $\sin(x) < 0$, what is $\cos(x)$?

↳ Requested Review

For more problems of this type see: 5.2.63 – 5.2.70

↳ Want to apply $\sin^2 + \cos^2 = 1$

$\Rightarrow \tan^2(x) + 1 = \sec^2(x) = \dfrac{1}{\cos^2(x)}$

$\left(\tan = \dfrac{1}{\cot(x)} \right)$

$\Rightarrow 5 = 1/\cos^2(x)$

$\Rightarrow \cos^2(x) = 1/5$

$\Rightarrow \cos(x) = \pm 1/\sqrt{5}$

Note $\cot(x) > 0$, but $\cos(x)/\sin(x) > 0$

$\Rightarrow \cos(x) < 0$

$\Rightarrow \cos(x) = -1/\sqrt{5}$.

Section 7.5.

Ex: Multiple angle trig eqns.

1) $\sin(5x) = \sqrt{2}/2$. find all solns.

period $= 2\pi/5$

① Solve $\sin(y) = \sqrt{2}/2$ (sub. $y = 5x$).

$y = \pi/4 + 2\pi n$ $\qquad x = \dfrac{y}{5}$

$y = 3\pi/4 + 2\pi n$.

② Solve for x from y.

$y = \pi/4 \rightsquigarrow x = \pi/20$.

Check: $\sin(5(\pi/20)) = \sin(\pi/4) = \sqrt{2}/2$.

$y = \pi/4 + 2\pi n \rightsquigarrow x = \pi/20 + \dfrac{2\pi}{5} n$.

$x = 3\pi/20 + \dfrac{2\pi}{5} n$.

2) $\sin(4x) = 1$. Solve for x.

$x = \pi/8 + \frac{\pi}{2} n$

① $y = 4x \rightsquigarrow$ Solve $\sin(y) = 1$

$y = \pi/2 + 2\pi \cdot n$.

② Solve for x using y. $x = \frac{y}{4}$

$x = \pi/8 + (\pi/2) \cdot n$.

Ex: $2 - 2\cos(x) = \sin^2(x)$.

$\cos^2(x) + \sin^2(x) = 1$.

Strategy: Put everything in terms of one trig fcn.

Use id $\cos^2(x) + \sin^2(x) = 1$.

$\Rightarrow 2 - 2\cos(x) = 1 - \cos^2(x)$.

$\Rightarrow \cos^2(x) - 2\cos(x) + 1 = 0$.

$y = \cos(x) \longleftrightarrow y^2 - 2y + 1 = 0$

$(y-1)^2 = 0$.

$\Rightarrow y = 1$

$\cos(x) = y = 1$

Solve for all x in $[0, 2\pi)$.

$x = 0$.

We add mult. of 2π to get all solns.

$x = 2\pi n$.

Ex: $\tan(x) = 3\cot(x)$.

Hint: Write everything in terms of sin.

$\leadsto \frac{\sin(x)}{\cos(x)} = \frac{3\cos(x)}{\sin(x)} \Rightarrow$

$$\sin^2(x) = 3\cos^2(x).$$

$$= 3(1-\sin^2(x)) \quad (\cos^2 x + \sin^2 x = 1)$$

$$= 3 - 3\sin^2(x).$$

$$\Rightarrow \quad 4\sin^2(x) = 3$$

$$\Rightarrow \quad \sin(x) = \pm\sqrt{3}/2.$$

$y = \sin(x).$

$$y^2 = 3/4$$

$$y = \pm\sqrt{3}/2.$$

Solve $\sin(x) = +\sqrt{3}/2$, and $\sin(x) = -\sqrt{3}/2$.

Ex: $\frac{1}{2}\cos(2x) - \sin(x) = \frac{1}{2}.$

$$\Rightarrow \frac{1}{2}\left(2\cos^2(x) - 1\right) - \sin(x) = \frac{1}{2}$$

Double angle formula

$$\Rightarrow \frac{1}{2}\left(2(1-\sin^2(x)) - 1\right) - \sin(x) = \frac{1}{2}$$

$\sin^2 + \cos^2 = 1$

$$\Rightarrow \quad 1 - \sin^2(x) - \frac{1}{2} - \sin(x) = \frac{1}{2}$$

$$\Rightarrow \quad \cancel{1} - \sin^2(x) - \sin(x) = \cancel{1}$$

$$\Rightarrow \quad \sin^2(x) + \sin(x) = 0$$

$$y^2 + y = 0$$

$$\Rightarrow \quad \sin(x)(\sin(x)+1) = 0$$

$$y(y+1) = 0$$

Solve $\sin(x) = 0$ and $\sin(x) + 1 = 0$.

Topic 1: Comp. the $\square$ and std form of quad.

Ex: $x^2 - 2x = -7$

$$x^2 - 2x + a = a - 7$$

St $x^2 - 2x + a = (x+b)^2$

$$(x+b)(x+b) = x^2 - 2x + a$$

Cross terms $2bx = -2x \Rightarrow b = -1$

$$\Rightarrow a = 1$$

$(x-1)^2 = x^2-2x+1 = -6$

$x-1 = \pm\sqrt{-6}$

$x = \pm\sqrt{-6}+1 = \pm i\sqrt{6}+1.$

Ex: $3x^2-8x=6$

$x^2 - 8x/3 = 2$

$x^2 - 8x/3 + a = a+2.$

$\quad \shortparallel$

$(x+b)(x+b)$

$b = -4/3$

$a = 16/9$

sub $\Rightarrow$ $x^2 - \frac{8x}{3} + \frac{16}{9} = \frac{16}{9}+2$

$\shortparallel$

$(x-4/3)(x-4/3)$

$\shortparallel$

$x^2 - \frac{4x}{3} - \frac{4x}{3} + \frac{16}{9}$

Ex: $x^2-2x+7 = f(x)$, put in std form

$f(x) = a\cdot(x-h)^2 + K.$ 3.4

Want $x^2-2x+\underbrace{a-a}_{0}+7$

so that $x^2-2x+a = (x-h)^2$

$\Rightarrow -a+7 = K.$

So by previous ex, $h=1$, $a=1$

$x^2-2x+1-1+7$

$(x-1)^2+6 = f(x)$

$x^2-x-x+1+6 = x^2-2x+7$ ✓

Add. Practice: 1.5.57 – 1.5.64 comp. □ to solve eqn

3.1.9 – 3.1.24 std form.